Footprint diameter for a cosmic-ray soil moisture probe: Theory and Monte Carlo simulations

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Abstract

We used a combination of diffusion theory and neutron transport simulations to estimate the lateral footprint for a cosmic-ray soil moisture probe. The footprint is radial and can be described by an exponential function. Our theory assumes, and our simulations confirm that the corresponding exponential folding length is closely related to the moderation length in air, which in this work is defined as the average net displacement experienced by neutrons while traveling from the point of emission from soil to the point of detection in air. These simulations indicate that the effective moderation length is 150 m at sea level, and that this value is fairly constant over a wide range of detection energies–from 10^0 to 10^5 eV. If we define the lateral footprint as the area encompassing two e-fold distances, i.e. the area from which 86% of the recorded neutrons originate, then the footprint diameter is nearly 600 m in dry air. Both theory and simulations indicate that the footprint is inversely proportional to air density and linearly proportional to the height of the sensor above the ground for heights up to 125 m. Futhermore, our simulations indicate that the dependence on soil moisture is small, but the dependence on atmospheric humidity is significant, with a decrease in the footprint diameter of 40 m for every 0.01 kg kg^{-1} increase in specific humidity. The good agreement between our theory and transport simulations suggests that the lateral footprint is determined mainly by the properties of air.

Key terms: soil moisture (1866); remote sensing (1855); hydrometeorology (1840); instruments and techniques: monitoring (1895); instruments and techniques: modeling (1894).

Key words: cosmic-ray probe, soil moisture, COSMOS, footprint, neutron, MCNP

1. Introduction

One of the major challenges in land-surface hydrology is measuring soil moisture at a scale commensurate with numerical models and other field measurements [*Bloschl*, 2001]. The significant spatial variability of soil moisture, even over a distance of just a few meters [e.g. *Western et al.*, 2002], makes it difficult to determine the average soil water content over a large area with conventional point sampling techniques—unless large numbers of samples are collected [e.g. *Jacobs et al.*, 2004]. On the other hand, satellite remote sensing of terrestrial microwave emissions can be used to infer spatially averaged soil moisture with global coverage, but the typical resolution of soil moisture data products is currently on the order of tens of kilometers—larger than is desired for many applications in hydrology and soil science. An alternative approach is the cosmic-ray technique [*Zreda et al.*, 2008; *Desilets et al.*, 2010], which when operated from a static position provides ground-based measurements at a scale intermediate between point and satellite observations. Operated in roving mode [*Desilets et al.*, 2010; *Zreda et al.*, 2012] the technique can provide regional and continental scale transects with sub-kilometer resolution.

The cosmic-ray method relies on the measurement of background neutrons emitted naturally from soil. These background neutrons are produced nearly continuously at Earth's surface through collisions between energetic cosmic-ray hadrons (mostly secondary neutrons) and terrestrial nuclei. The exceptional neutron scattering properties of hydrogen make the background neutron intensity highly sensitive to the presence of hydrogen-rich materials such as water [*Zreda et al.*, 2008]. Like a conventional "downhole" neutron probe [*Evett and Steiner*, 1995], which utilizes an artificial radioisotopic neutron source instead of natural background, the sensitivity of a cosmic-ray probe to water is mainly due to the low mass and relatively large elastic scattering cross section of the hydrogen nucleus (~4 barns at 1 MeV [*Chadwick et al.,* 1992]). The low mass of hydrogen means that momentum can be efficiently transferred between a neutron and the hydrogen nucleus during elastic collisions; the large elastic scattering cross section means that such collisions are highly probable when hydrogen is present. Consequently, soil moisture, a major source of hydrogen at the land surface, plays a dominant role in slowing down cosmic-ray neutrons near the land-air interface. For example, in a soil comprising only 3% water by mass, nearly half of the energy loss is caused by collisions with the hydrogen contained in soil water. These moderating collisions lower the intensity of neutrons in the fast to epithermal energy range, which explains the observed anticorrelation between the neutron counting rate and soil water content observed in field experiments [*Zreda et al.*, 2008].

The radius of influence for a cosmic ray probe is governed by the same scattering laws that apply to a conventional neutron probe. But there are important differences in the source-detector geometry that give the cosmic-ray probe a much larger footprint. With the subsurface probe, the detector records neutrons that are scattered back from a point source that is co-located with the detector. But with the cosmic-ray probe, the neutrons are emitted from a laterally extensive plane source (the soil surface) and received by a detector that is located some distance above this plane source. With the conventional neutron probe, nearly all of the scattering takes place within a radius of tens of centimeters around the probe. But with the cosmic-ray probe, scattering in air allows neutrons to travel hundreds of meters from where they originate.

The application of subaerial cosmic-ray neutron fluxes to large-scale soil moisture determinations is a fairly recent development, in contrast to the decades-old technique of lowering radioisotopic neutron sources into boreholes to measure water content or porosity. Although the underlying neutron scattering principles are similar between the two methods, one important distinguishing characteristic of the (non-invasive) cosmic-ray technique is that a large fraction of the counted neutrons are scattered in the air before reaching the detector. With the cosmic-ray technique the scattering properties of air should therefore play an important or even dominant role in determining the footprint diameter. This diameter should be related to the distance that a neutron travels on average between where it originates or is emitted from soil to where it is detected milliseconds later above the ground by a cosmic-ray moisture probe. Given that (1) the neutron source is distributed laterally across a large area; (2) the collision mean free path in air is on the order of several tens of meters or more depending on energy (see section 4.5) ; and (3) that multiple collisions in air are likely to occur, the footprint diameter should be on the order of many tens to hundreds of meters. Because of this large scale, it is difficult to conduct field experiments that can be used to precisely quantify the scale and its dependence on detector parameters and environmental variables. For that reason we use a combination of neutron diffusion theory and neutron transport simulations to quantify the footprint diameter for a cosmic-ray soil moisture probe.

We first develop an analytic framework from diffusion theory. We use this theory to expresses the radius of influence in terms of the mean atmospheric travel distance between emission from soil and detection in the atmosphere. This distance is termed the moderation length. The derived relationship for the radius of influence is first parameterized using a semi-theoretical calculation, which involves gross simplifications of neutron scattering physics, and then more realistically using Monte Carlo neutron transport simulations. Theoretical relationships and numerical simulations are used to illuminate the dependence of the moderation length on soil moisture, neutron energy, barometric pressure and height above the ground. The footprint diameter corresponding to 86% of the source area is then calculated as a multiple of the moderation length.

2. Theory

2.1. Radius of influence defined

We define the area of influence (lateral footprint) as a radially symmetric region contributing a specific, arbitrarily large fraction of the counting rate to a centrally located sensor. The radius of influence is therefore the length circumscribing that area. In the following analysis we use elementary diffusion theory to show that a planar, uniform neutron source has a radius of influence that can be described by a single parameter, an exponential-folding (*e*-folding) length. One *e*-folding length is equivalent to the radius encompassing the source area contributing 100 x $(1-e^{-1})$% of the counts. The numerical value of this *e*-folding length can be calculated directly from the bulk nuclear properties of the scattering medium and diffusion theory (section 2.3) or inferred from neutron transport simulations (section 3).

To develop an expression for the area of influence, we begin by assuming that any volume or area neutron source can be interpreted as being made up of a large number of point sources. The total neutron flux at any particular point in a domain can then be determined from the superposition of the fluxes from each contributing point source [*Glasstone and Edlund*, 1952]. We use this principle (principle of superposition) as the basis for calculating the flux distribution around a point source, and use a collection of point sources to define a ring source. Finally, we integrate concentric rings to obtain the flux to a point at the center of the ring as a function of radial distance.

According to diffusion theory, the neutron flux ϕ [n L^{-2} T^{-1}] surrounding a point source in an infinite, homogenous medium, varies with distance r [L] according to

$$\phi(r) = \frac{S_{\mathrm{pt}}}{4\pi D}\frac{e^{-\frac{r}{L}}}{r^n} \qquad 1$$

where S_{pt} is the source intensity [n T^{-1}], D is the diffusion constant [L^{-1}] and L is the e-folding length [L] that describes attenuation due to nuclear absorption (note that L is a dimension; L is a variable). Nuclear absorption is the process in which a neutron collides with a nucleus and is captured by it, resulting in the formation of a heavier nucleus and a reduction of the neutron flux. The expression $\exp(-r/L)$ accounts for nuclear absorption by the constituents of air, and the factor $1/r^n$ accounts for radial spreading of the flux from the point source. For neutrons behaving diffusively, where the loss of outgoing neutrons is partially compensated by a backward scattered flux, n = 1 [*Glasstone and Edlund,* 1952].

The intensity of neutrons emitted from an infinitely thin annulus, S_{ring} [n T^{-1}], is given by

$$S_{\mathrm{ring}} = S_{\mathrm{pt}} 2\pi r \qquad 2$$

and therefore the neutron flux at the center of the ring, ϕ_0, is

$$\phi_0(r) = \frac{S_{\mathrm{ring}}}{4\pi D}\frac{e^{-\frac{r}{L}}}{r} = \frac{S_{\mathrm{pt}}}{2D}e^{-\frac{r}{L}} \qquad 3$$

which can be rewritten as

$$\phi_0(r) = C\mathrm{e}^{-\frac{r}{L}} \quad 4$$

where $C = \frac{S_{\mathrm{pt}}}{2D}$. 5

To obtain the flux at any point located on the surface of an infinite plane source, equation 4 can be integrated over a continuum of rings extending to infinity,

$$\int_0^\infty \phi_0(r)dr = C\int_0^\infty \mathrm{e}^{-\frac{r}{L}} dr = C \cdot L \quad 6$$

and the intensity of neutrons originating within distance $r = L$ is then given by

$$\int_0^L \phi_0 = C\int_0^L \mathrm{e}^{-\frac{r}{L}} dr = C \cdot L\left(1 - \mathrm{e}^{-1}\right) \quad 7$$

The probability that a neutron originates from within $r = L$ is then given by

$$\frac{\int_0^L \phi_0(r)dr}{\int_0^\infty \phi_0(r)dr} = 1 - \mathrm{e}^{-1} \quad 8$$

which shows that the e-folding length L is equivalent to the radius circumscribing an area contributing 100 x [1-e^{-1}]% of the total flux to a point detector located on an infinite plane source. Taking advantage of the exponential law that defines the radial distribution of neutrons, we now define the footprint diameter as the distance corresponding to two e-fold increases in the contribution to the count rate, i.e., the diameter of a circle encompassing the source area for 86% of the recorded neutrons. The analysis above demonstrates that this diameter is equivalent to 4L. We proceed by quantifying the value of L in the atmosphere.

2.2. Radius of influence defined as a moderation length

The main challenge in applying the theoretical relations in the previous section is in quantifying the e-folding length, L. According to diffusion theory, the value of L for thermal neutrons can be calculated directly from the elemental abundances of neutron absorbers that are present in the diffusing medium. But in most earth materials, including the atmosphere, absorption is significant only at energies below ~1 eV, whereas the detectors that are typically used for soil moisture determinations [e.g. *Zreda et al*., 2012] are sensitive mainly to energies >1 eV (fast to epithermal range). At these higher energies, elastic interactions resulting in incremental energy loss are much more probable than absorption of the neutron by the scattering medium. In this case neutron intensity is controlled by the moderating properties of the medium, and slowing down theory must be employed.

Moderation is analogous to absorption in that both processes reduce the flux of the neutrons over a specific energy range. A neutron having an initial energy within some arbitrarily defined energy bin can be considered absorbed (removed from the bin) when the energy has been reduced to below the lower threshold of the bin. As with absorption, the probability of neutron survival at a given energy (i.e. the transmission over some distance without significant energy loss) diminishes exponentially with the distance traveled. To determine the value of the corresponding *e*-folding length, we take advantage of the fact that the *e*-folding length is by definition equivalent to the mean distance (mean free path) traveled by a neutron before being absorbed [*Glasstone and Edlund*, 1952]. For neutrons undergoing moderation, L can be taken as the mean distance between the point of emission and the point at which the energy of the particle is reduced below a specific energy threshold. The relevant distance for a cosmic-ray probe can be determined from particle tracking simulations, as described in section 3, or by calculating analytically the mean distance traveled by a neutron from the point of emission from soil to the point of detection within some volume of air above the ground, as in the following section.

2.3. Moderation length calculated for an infinite, homogenous atmosphere

The radius of influence for a cosmic-ray probe located above the land surface can be described by the mean distance traveled between emission and moderation to detection energy. We refer to this distance as the moderation length L_{m} and emphasize that our definition of moderation length is more general than the one conventional to neutron diffusion theory. In conventional usage, the moderation length (also called slowing-down length) is given as the average net displacement between emission and moderation to thermal energy (i.e. to equilibrium with thermal vibrations of surrounding molecules) [*Glasstone and Edlund*, 1952]. But in this paper the moderation length is defined as the average net displacement as a neutron is moderated from an initial energy E_i, the energy of emission, to final energy E_f, the energy of detection. In reality, neutrons are usually recorded over a wide range of energies, but for analytical convenience we assume in this section that E_f corresponds to the median sensitivity of the detector.

The moderation length is calculated from the root-mean-square displacement $\langle r^2 \rangle$ of a neutron while being slowing from E_i to E_f. This is given by

$$\langle r^2 \rangle = \left(\lambda\sqrt{n}\right)^2 \qquad 9$$

where λ is the mean distance between collisions (jump length) and n is the number of collisions. The average number of collisions required to slow a neutron from E_i to E_f is given by

$$n = \frac{\eta}{\xi} \qquad 10$$

where η is the number of log decrements between E_i to E_f,

$$\eta = \ln E_i - \ln E_f \qquad 11$$

and ξ is the log decrement energy loss per collision, which according to the law of conservation of momentum is given by

$$\xi = 1 + \frac{(A-1)^2}{2A} \ln \frac{A-1}{A+1} \qquad 12$$

[*Glasstone and Edlund*, 1952]. The effective atomic weight for a mixture of elements is

$$A_{\text{eff}} = \frac{\sum_{i=1}^{n} A_i \lambda_i^{-1}}{\sum_{i=1}^{n} \lambda_i^{-1}} \qquad 13$$

Where λ_i is the average jump length for element *i*. The value for air is A_{eff} = 14.25 g mol^{-1}, which yields η = 7.5 elastic collisions on average to reduce the neutron energy by one logarithmic decrement in air. The jump length is equivalent to the collision mean free path, given as

$$\lambda = \frac{1}{\Sigma_s} \qquad 14$$

where Σ_s, the macroscopic scattering cross section is calculated from

$$\Sigma_s = \sigma_s \frac{A}{\rho N} \qquad 15$$

and A is atomic mass, ρ is material density, N is Avogadro's number and σ_j [cm^2] is the (microscopic) elastic scattering cross section for reaction *j*. Elastic scattering cross sections can be found in published tables of nuclear data (e.g *Chadwick et al.*, 2006). For a mixture of elements with $i = n$ components

$$\lambda = \sum_{i=1}^{n} \sigma_{s,i} \frac{A_i}{\rho_i N_i} \tag{16}$$

The moderation length is then calculated from the mean square displacement (Equation 9) using the relation [*Glasstone and Edlund*, 1952]

$$L_{\mathrm{m}}^2 = \frac{1}{6}\left\langle r^2 \right\rangle \tag{17}$$

For improved accuracy, there are two additional complexities that can be incorporated into the calculation of the moderation length. One is the decrease with energy in the elastic scattering cross section. This decrease means that the jump length increases with increasing energy, as shown in Figure 1. To account for this strong energy dependence we calculate average values for the jump length over logarithmically spaced energy bins. If the effective jump length for energy bin *i* is given by the logarithmically-weighted average value λ_i, where $i = 1$ corresponds to the first log decrement of energy loss after neutron emission, then the total root-mean-square displacement over all log decrements is estimated to be:

$$\left\langle r^2 \right\rangle = \sum_{\mathrm{i}} \lambda_i \left(\sqrt{i} - \sqrt{i-1}\right)\sqrt{n_i} \tag{18}$$

Because the average number of collisions experienced by a neutron between emission and detection increases with initial energy, the moderation length also increases with increasing initial energy (assuming the final energy is fixed). This effect is compounded by the tendency for the collision mean free path in air to increase with energy.

A second complexity thus far neglected is that neutrons are emitted from soil over a range of energies. This is because cosmic-ray neutrons are emitted from nuclei with initial energies governed by the nuclear evaporation process, which is statistical in nature, and because neutrons generated at depth lose energy stochastically through elastic collisions as they travel through soil. If the final (detection) energy, E_f, is assumed to be 10^2 eV, which is approximately the median value for a slow neutron detector moderated with 1-3 cm of plastic, and the initial energy is assumed to range from the evaporation peak at 2×10^6 eV [*Goldhagen et al.*, 2004] down to the detection energy, then the moderation length will range from approximately 240 m for neutrons emitted at 2×10^6 eV to 0 m for neutrons already at the final energy, E_f. The effective moderation length, averaged over all neutrons emitted from soil, should lie somewhere between these two values, with the precise value depending on the shape of the soil emission spectrum. It is therefore necessary to calculate the emission spectrum with reasonable accuracy.

We used the neutron transport code Monte Carlo N-Particle eXtended (MCNPX) [*Pelowitz,* 2005] to determine the emission spectrum for neutrons emitted from a dry SiO_2 soil. Our model utilizes a source function that decreases with depth in soil according to an *e*-folding length of 150 g cm^{-2}, and follows a nuclear evaporation energy spectrum peaked at 2×10^6 eV. The flux of neutrons exiting the soil surface is tallied in a 2-meter thick slab directly overlying the soil. The atmosphere is modeled as a vacuum to prevent neutrons from diffusing back into the detector layer after passing into atmosphere above. Our results (Figure 2) indicate that at energies above 10^4 eV the differential energy spectrum is not yet in equilibrium, as can be seen from the deviation of our simulated spectrum from the 1/*E* spectrum predicted for neutrons in slowing down-equilibrium [*Glasstone and Edlund*, 1952]. Deviation from the theoretical $1/E$ spectrum has also been observed in simulations and measurements of the secondary cosmic-ray neutron spectrum performed by others. In fact, the cosmic-ray neutron spectrum always shows a bump in the 1-3 MeV region [*Goldhagen et al.*, 2004], which is explained by the increased contribution of evaporation emissions over a relatively narrow range of energies. The implication of this behavior is that the proportion of fast neutrons to slow neutrons is significantly greater than would be predicted by a simple equilibrium slowing-down spectrum. The spectrum from our transport simulations reflects more accurately the non-equilibrium phenomenon.

Using our simulated spectrum, the average moderation length is calculated as the weighted-average

$$\langle L_{\mathrm{m}} \rangle = \frac{\sum_k L_{\mathrm{m},k} w_k}{\sum_k w_k} \qquad 19$$

where $L_{\mathrm{m},k}$ is the moderation length for a neutron with initial energy E_k and the weights w_k are given by

$$w_k(E) = \phi_{E_k} \left(\ln(E_k) - \ln(E_{k-1}) \right) \qquad 20$$

where ϕ_{E_k} is the logarithmically averaged differential neutron flux at energy E_i. Using the corrected spectrum, the moderation length from $E_i = 2\text{x}10^6$ eV to $E_f = 10^2$ eV is 147 m. This length is equivalent to the average crow-flight distance traveled in the atmosphere by a cosmic-ray neutron from the point of emission to the point of detection at 10^2 eV. Note that up to this point we have assumed a dry atmosphere at sea level. The effects of atmospheric humidity and elevation on the moderation length are significant but will not be discussed until section 4. Unless otherwise stated, L_m in this paper should always be assumed to represent the dry air value at sea level.

The analytical theory outlined above has several important implications. First, it suggests that because the footprint is directly proportional to the collision mean free path in air, the footprint should increase with increasing elevation (decreasing atmospheric density). Second, it implies that the footprint should increase with increased separation between the energy of emission and energy of detection due to the larger number of collisions that would be required to moderate the neutron. Third, because the radius of influence is assumed to grow in proportion to the square root of the number of collisions, it implies that the first few collisions in air are much more important to net displacement than later ones. This in turn suggests that small changes in energy sensitivity of the detector may have little effect on the footprint radius. And fourth, if our underlying assumption that neutron transport is dominated by the properties of the atmosphere is correct, then the radius of influence should be insensitive to soil moisture so long as the shape of the source spectrum isn't much affected. In the next section these dependencies of the lateral footprint are investigated more rigorously using full Monte Carlo transport simulations, and by more accurately modeling the scattering interactions in both land and air.

3. Monte Carlo transport simulations

We used the code MCNPX to overcome limitations inherent to diffusion theory. With MCNPX, individual neutron scattering histories are simulated one-at-a-time by sampling probability distributions that describe the initial source energy, source direction, and likelihood of a particular nuclear interaction in a material of prescribed composition. General advantages of the Monte Carlo approach over analytical methods include better accuracy near boundaries, continuous-energy treatment of nuclear interactions, and flexible tallying options. A major advantage of MCNPX for this work is its ability to record the coordinates of individual scattering interactions for a large number of simulated particle histories.

In these simulations it is necessary to tally fluxes over a finite energy range. We have chosen as the upper edge of this range $E_{f,u}= 10^3$ eV and as the lower edge $E_{f,l} = 10^1$ eV. Assuming that the differential neutron intensity drops off in proportion to $1/E$, as is indicated by slowing down theory (Glasstone and Edlund, 1956), this means that the flux-weighted mean energy is 10^2 eV, which is the same as E_f in the previous section. So for these simulation the mean final energy is the same as E_f assumed in section 2.3.

For the sake of computational efficiency, we make the assumption that a spatially distributed source can be treated as a collection of point sources, as in section 2. This allows us to track a large number of particles emanating from a single point source. A collection of point sources can then be integrated over a circle to simulate a ring source, and then integrated once again over some radial distance to simulate a plane source covering a disc of finite-diameter. Computationally, this approach is vastly more efficient (approximately a factor of 10^6 improved processing time) than the alternative of simulating a point or small volume detector located in the center of an aerially expansive plane source (in which case only a small fraction of the particles emitted from the plane will ever reach the detector). As demonstrated in section 2, the mean distance traveled by a neutron as it is moderated from E_i to E_f is equivalent to the (one e-folding length) radius of influence for a point detector sensitive to neutrons at E_f. Although diffusion theory indicates that these two distances (moderation length and radius of influence) are described by the same *e*-folding length, it does not provide an easy way to calculate that length in the presence of the air-ground interface.

From our Monte Carlo simulations we calculate L_m as the mean crow-flight distance traveled by each neutron from the point of emission to the point of detection. We assume that a neutron will be emitted from soil at an initial energy E_i but must be moderated to average energy E_f in order to be detected. As in our semi-theoretical model, the separation between E_i and E_f matters because each additional random collision will tend to carry a neutron farther afield from the point of emission. In our simulations we specify that in order to be detected the neutron must also be within two meters of the land surface, which is where our detectors are typically installed for static operations [e.g. *Zreda et al.,* 2008; *Desilets et al.,* 2010; *Zreda et al.*, 2012]. Neutrons that are within this two meters thick "detector layer" but have not crossed the threshold E'_u are not tallied.

The energy of emission from soil is not directly specified in our model. It is instead determined by sampling an evaporation source spectrum and then stochastically transporting neutrons through the top 0-4 m of soil. The source is specified as occurring on a line that extends along the *z*-axis from the land surface to the bottom boundary of the domain, a distance of 4 m. To simulate the attenuation of the neutron-producing cosmic ray flux (source function) as it penetrates the subsurface, the source intensity in our model decreases exponentially with depth according to an *e*-folding length of 150 g cm^{-2}. The initial energy for neutrons originating along this line is sampled randomly from an evaporation spectrum having a central energy of 2x10^{6} eV, which corresponds approximately to the observed peak evaporation energy for cosmic-ray neutrons [*Goldhagen et al.*, 2004]. We found that neutrons are emitted from the ground within a lateral radius of only 0.5 m from the line source on average. Given that the length scale for transport in the air is on the order of at least 100 m, this means that the source emission area can be considered to be a point for most practical purposes.

Our model domain is a box with width and length of 2000 m and height of 7645 m. The subsurface is 4 meters deep and is modeled as pure SiO_2 with a bulk density of 1.4 g cm^{-3}. The atmosphere is 7641 meters thick and contains 78% N and 22% O with a density that ranges from $1.2x10^{-3}$ g cm^{-3} at the ground surface to $0.6x10^{-3}$ g cm^{-3} at the top. We ran at least 200,000 particles per simulation, of which typically 2-3% meet the criteria for detection. Most of the remaining 97-98% were either transported to heights greater than 2 m above the ground or to some depth below the ground by the time they had been moderated to within the range E'_u to E'_l.

Simulations were performed at varying soil moisture contents, air densities, specific humidities and for different heights of the detector layer above the ground. To simulate a detector located at a position well above the ground surface, fluxes were tallied in a 2 m detector layer centered at the selected altitude. We tracked the coordinates of all particles that passed through the detector cell with energies within the specified criteria ($E_{f,u}>E>E_{f,l}$). These particles are referred to as detected neutrons, and the particle coordinates at the point of entry into the detector layer define the point of detection.

4. Results and discussion

We used the particle tracking feature in MCNPX to record the collision coordinates of all particles meeting the detection criteria. Example particle tracks for three arbitrarily selected random walks are shown in Figure 3. Tracks A and B show that numerous collisions can occur in air before a neutron is detected near the ground. Trajectories A and C demonstrate that a track may intersect the ground a second time after a neutron is first emitted. A and C also illustrate the tendency for the first few collisions in air to account for most of the total displacement, as implied by diffusion theory (Equation 9), they also show the tendency for the collision mean free path to decrease with additional collisions.

We calculated the radius of influence from our Monte Carlo simulations according to three different methods. First, we calculated the mean radial displacement for all “detected” particles. The radial displacement is the net radial distance traveled from the source to where an entry is made into the detector layer at energy $E'_u > E > E'_l$. This distance gives the total displacement of neutrons but does not take into account secondary interactions with the ground that might occur in transit. Our simulations indicate that a neutron intersects the ground on average 2.0 times before being detected. Such interactions with the ground would result in additional moderation, and therefore could influence the distance weighting function. For example if those intersections tended to occur nearer (or farther) from the detector in comparison to the point of emission, then the footprint should shrink (or increase) somewhat in comparison to the net displacement. To account for this we calculated the radius of influence a second way, instead as the mean distance to each collision in the detector layer for neutrons of any energy. We assume that the collision density at any point should be directly proportional to the neutron flux. Because collisions in the detector layer should be a good proxy for collisions in the nearby ground, this approach should intrinsically be sensitive to secondary interactions in soil following emission. However, unlike the first method, this method does not track the final coordinates corresponding to where the neutron is detected. Finally, we calculated the radius of influence from the mean distance to the final collision in the detector layer for all “detected” neutrons.

The three methods yield effective moderation lengths for dry land at sea level that are within 1% of the value of 150 m. Although secondary interactions with the ground are apparently common, the agreement between the three methods suggests that they do not much affect the radius of influence. One explanation is that the distance between each secondary soil interactions and the point of detection (or final interaction) is on average equal to the distance from the point of detection to the point of emission. In other words, there is little net gain or loss in lateral displacement caused by the on average 2.0 additional collisions with the ground.

The effective value of 150 m derived from transport simulations agrees well with the theoretical value derived in section 2. This simulated value is valid for $E_{f,u}$=10^3 eV and $E_{f,l}$=10^1 eV. Assuming $E_f = 10^2$ eV, the theoretical moderation length (using the modeled emission spectrum) is 147 m in air. The good agreement between theory and simulations confirms that the radius of influence is determined mainly by the properties of air, at least when vegetation is absent. Specific variables influencing the moderation length are described in the following subsections.

4.1. Dependence on soil moisture

Our simulations indicate that the moderation length is only about 5% smaller over saturated soil (L_m=142 cm) compared to dry land (L_m=150). The direction of the change is consistent with the suggestion that secondary interactions with wetter ground rapidly moderate neutrons thereby limiting their range, making the footprint smaller. The small dependency on soil moisture is again consistent with an effective moderation length that is determined mainly by the transport properties of the atmosphere. We therefore conclude that this effect can be ignored.

4.2. Dependence on energy

We calculated the radius of influence over dry soil as a function of detection energy (Figure 4). In agreement with theory, the footprint tends to increase with the separation between the detection and emission energies. However, the radius of influence apparently decreases with energy at the lowest energies (10^{-2} to 10^{-1} eV range). Neutron absorption likely plays a role in explaining this behavior. At such low energies (<1 eV), the probability of nuclear absorption by atmospheric nitrogen becomes significant. This may favor the survival of neutrons that are moderated in the soil relatively close to the detector because the remaining distance to the detector (and therefore likelihood of absorption while transiting the atmosphere) would be minimized. We conjecture that those neutrons moderated in the ground close to the detector may also have originated relatively closer to the detector, which would shrink the radius of influence.

Although the moderation length tends to increase with decreasing detection energy, the dependence is apparently small over a broad plateau from 10^0 to 10^5 eV. This indicates that differences in detector energy sensitivity will not significantly change the footprint. It should therefore be possible to use sensors with different energy sensitivities to monitor the same sample area, so long as the sensors are mainly sensitive to neutrons in the range 10^0 and 10^5 eV.

4.3. Dependence on atmospheric pressure

Because the scattering mean free path in air depends inversely on air density, the moderation length should increase as atmospheric pressure decreases. If the moderation length is governed almost entirely by the properties of air, which the agreement between theory and simulations suggest is the case, then the radius of influence should be inversely proportional to atmospheric pressure. The moderation length, L_m at pressure P is then given as:

$$L_m = L_{m,0}\left(\frac{P_0}{P}\right) \quad 21$$

where $L_{m,0}$ and P_0 are reference values. To test the validity of this equation, we calculated the moderation length for different atmospheric densities using MCNPX (Figure 5). Our transport simulations indicate that Equation 21 is accurate, and further support the observation that the properties of air dominate the effective moderation length. Note that atmospheric pressure depends mainly on elevation, but variations of 10-20 mb at any given location are quite common over the course of a month due to atmospheric circulation. A 20 mb change in pressure at sea level (1013 mb) would mean only a 2% change in L_m.

4.4. Dependence on atmospheric humidity

We have until now assumed that the atmosphere is completely devoid of humidity in calculating L_m. In reality, the specific humidity of the atmosphere can be in excess of 0.02 kg kg^{-1} in tropical regions (Laing and Evans, 2010). Increasing the partial pressure of water vapor has an effect similar to increasing the total pressure; however water vapor is much more effective at shortening the moderation length on a per mole basis because of the presence of hydrogen. We simulated this effect in MCNPX by changing the specific humidity (q) uniformly across the entire atmosphere. We express our results as the ratio of the wet atmosphere moderation length to the dry atmosphere moderation length at sea level (Figure 6). These results indicate a nearly linear relationship between the ratio of wet to dry moderation lengths and q, with a coefficient -7.76 m/(kg kg^{-1} H_20). This translates to a decrease in the sea level footprint diameter of nearly 40 m for every 0.01 kg kg^{-1} increase in specific humidity.

4.5. Dependence on height above ground

Diffusion theory indicates that the radius of influence should increase with height above the ground. A simple geometric relationship can be used to approximate this effect. We begin with equation 4 and substitute L_m for L, i.e.

$$\phi_0(r) = C\mathrm{e}^{-\frac{r}{L_\mathrm{m}}}$$

Now consider that when a detector is raised to some height h above the ground, all neutrons emanating from the ground must travel farther to reach the detector. This leads to increased attenuation of all neutrons emanating from the ground, but the amount by which the flux is reduced depends on the radial location. The relative increase in the distance between a point on the ground and the detector decreases as one moves away from the detector, where at large distances there is relatively no change in the path length (i.e. the aspect ratio approaches infinity). Given that the intensity of near-field neutrons will drop more rapidly than the intensity of far-field neutrons, the radius encompassing a given percentage of the source neutrons must expand as the height of the detector is increased.

Because the effect is largely geometric, the problem can be approached analytically. We define L^* as the radius of influence, traced on the ground, for a detector placed at a height h above the ground. To be consistent with our previous definition of L, the circle circumscribed by L^* should contain $1-e^{-1}$ of the counts. If neutrons are emitted from a source located on the ground at radius r from the origin, and the detector is located at height h above the origin, then the mean crow-flight distance between the source and detector is simply the hypotenuse of a right triangle (Figure 7), which we will call x.

By analogy to equation 3, the flux contributing to a detector located above the center of the ring (recalling that the ring traces along the ground) is

$$\phi_0(r) = \frac{S_{\mathrm{ring}}}{2D}\frac{\mathrm{e}^{-\frac{x(r)}{L_m}}}{x(r)} = \frac{Cr}{x(r)}\mathrm{e}^{-\frac{x(r)}{L_m}} \qquad 22$$

where $x(r) = \sqrt{r^2 + h^2}$. By definition, L^* must satisfy the equation

$$\frac{\int_0^{L^*}\phi_0(r)dr}{\int_0^{\infty}\phi_0(r)dr} = 1 - \mathrm{e}^{-1} \qquad 23$$

which can also be written as

$$\frac{\int_0^{L^*}\frac{r}{\sqrt{r^2+h^2}}\exp\left(-\frac{\sqrt{r^2+h^2}}{L_m}\right)dr}{\int_0^{\infty}\frac{r}{\sqrt{r^2+h^2}}\exp\left(-\frac{\sqrt{r^2+h^2}}{L_m}\right)dr} = 1 - \mathrm{e}^{-1} \qquad 24$$

which has the solution

$$\frac{-\frac{1}{L_m}\left[\exp\left(-\frac{\sqrt{r^2+h^2}}{L_m}\right)\right]_o^{L^*}}{-\frac{1}{L_m}\left[\exp\left(-\frac{\sqrt{r^2+h^2}}{L_m}\right)\right]_o^{\infty}} = 1 - \mathrm{e}^{-1} \qquad 25$$

This can be simplified to

$$1\text{-}\exp\left(-\frac{\sqrt{L^{*2}+h^2}-h}{L_m}\right)=1-\mathrm{e}^{-1} \qquad 26$$

where equality holds only if

$$\sqrt{L^{*2}+h^2}-h=L_m \qquad 27$$

Solving for L^* then yields an expression for the radius of influence given in terms of the detector height and moderation length,

$$L^*=\sqrt{(L_m+h)^2-h^2} \qquad 28$$

It can also be seen that the hypotenuse figure x must be equal to $L_m + h$ when $D = L^*$.

For the case in which h is on the order of L_m or smaller, the expression can be linearized:

$$L^*\approx L_m+h \qquad 29$$

It should be emphasized that due to attenuation by the atmosphere, there is a practical limit to the height at which the cosmic-ray technique can be used. The value $2L_m$ (~300 m) is a reasonable practical upper limit to assume. At this height the measured neutron flux should largely be uncorrelated to conditions on the ground since the flux of neutrons originating from the ground is reduced by 100 x e^{-2}% over the distance of two *e*-fold lengths. Therefore, while the sensitivity to soil neutrons decreases with sensor height, the diminished number of soil neutrons reaching sensor will tend to represent soil over a much larger area.

To test the validity of these equations we again relied on the particle tracking feature in MCNPX. The value of L^* for different heights was calculated as the mean distance from the point of emission to the point of detection, given a 2 m thick detector layer. The middle of the detector layer was located at heights above the ground ranging from 1 to 200 m. We tracked the coordinates of all collisions within the detector layer for particles that were eventually tallied in the detector layer. Our results (Figure 8) indicate that the radius of influence is described well by equation 28 up to a height of at least 200 m, and by the linearized version (equation 29) at least up to 125 m. In fact, the linearized version appears to be superior to equation 28 up to a height of 125 m, although the original equation appears to give the correct shape. We therefore conclude that for nearly all ground- and tower-based applications, the moderation length given by equation 29 should suffice for estimating the footprint diameter. We also hasten to point out that small differences in probe height (a few meters) should not influence the footprint diameter very much, at least for neutrons with energy >1 eV.

In separate simulations we also recorded the location of the first 20,000 atmospheric collisions for detected particles. Those results (Figure 9) help visualize the lateral range of detected neutrons in the air as a function of detector height. As the height of the detector layer is increased, the collision density near the ground decreases, implying that neutrons are less influenced by soil moisture. However, the neutron cloud also becomes more spread out with height, meaning that distant collisions are proportionately more important, and that the area of influence is greater.

5. Summary and conclusions

We used semi-analytic diffusion theory and Monte Carlo simulations to derive and parameterize expressions for the radius of influence for a cosmic-ray probe detecting neutrons in the 10^1-10^3 eV range. Our work indicates that:

(1) The radius of influence can be described by a single parameter: the exponential *e*-folding length L;

(2) The *e*-folding length that describes the slowing down of neutrons in air is the moderation length, L_m, The value of the moderation length for dry air is about 150 m at sea level for neutrons detected at energies of 10^2 eV. This gives a dry air footprint diameter of $D = 4\ L_m$ =600 m for the area contributing 86% of the counts;

(3) The footprint diameter depends mainly on the properties of air, and therefore is insensitive to soil water content;

(4) The footprint diameter increases in proportion to the square root of the average number of collisions required to moderate a neutron to detection energy;

(5) The footprint diameter is inversely proportional to air density and therefore increases with elevation;

(6) The footprint diameter at sea level decreases by 40 m for every 0.01 kg kg^{-1} increase in specific humidity.

(7) For heights of up to 100 m, the dependence of the footprint diameter on height is given accurately by $D(h) = 4(L_m + h)$, where h is the height above the ground in meters. This relation implies that precise positioning of the probe with height is not critical to the footprint diameter.

(8) The cosmic-ray technique should be usable up to a maximum sensor altitude of approximately $2L_m$ (i.e. 300 m).

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Figures

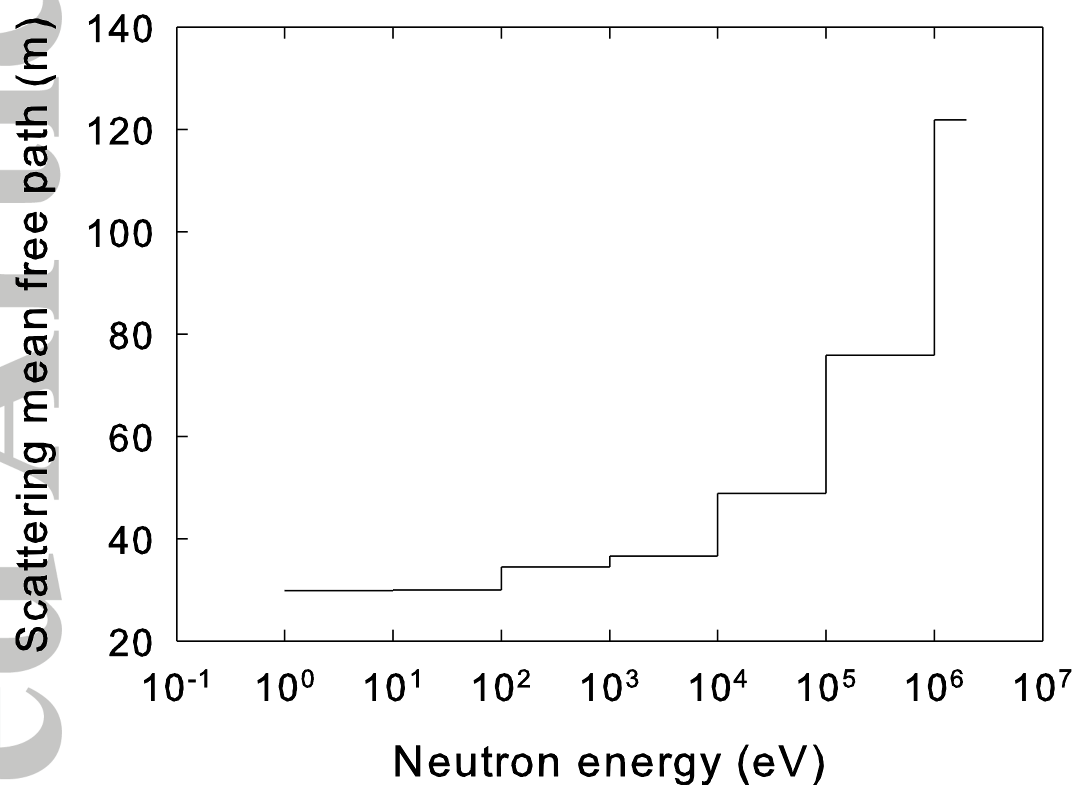


Figure 1. Neutron scattering mean free path in air at sea level as a function of energy according to elastic scattering cross sections for nitrogen and oxygen obtained from ENDF/B-VII.0 [*Chadwick et al.*, 2006] assuming an atmospheric density of 0.00122 g cm^{-3} and composition of 78% N and 22% O.

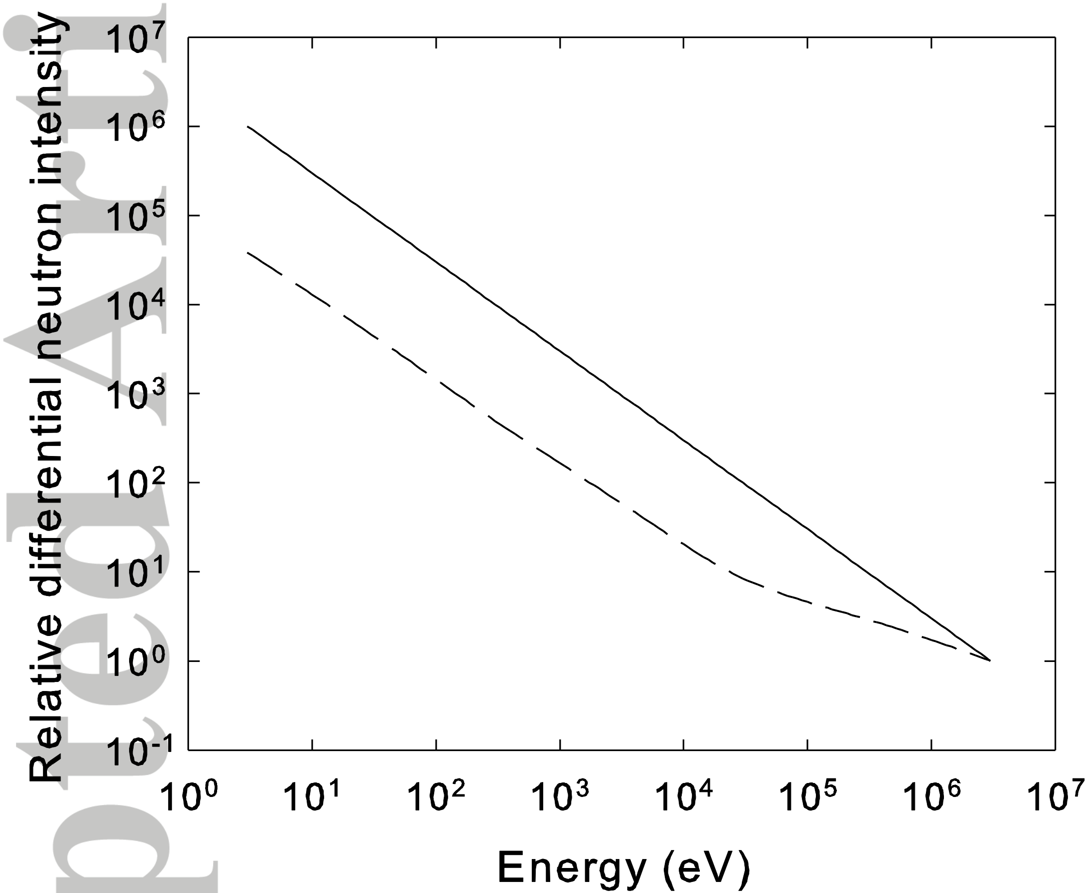


Figure 2. Soil emission spectrum simulated in MCNPX (dashed) compared to theoretical 1/*E* slowing down spectrum (solid).

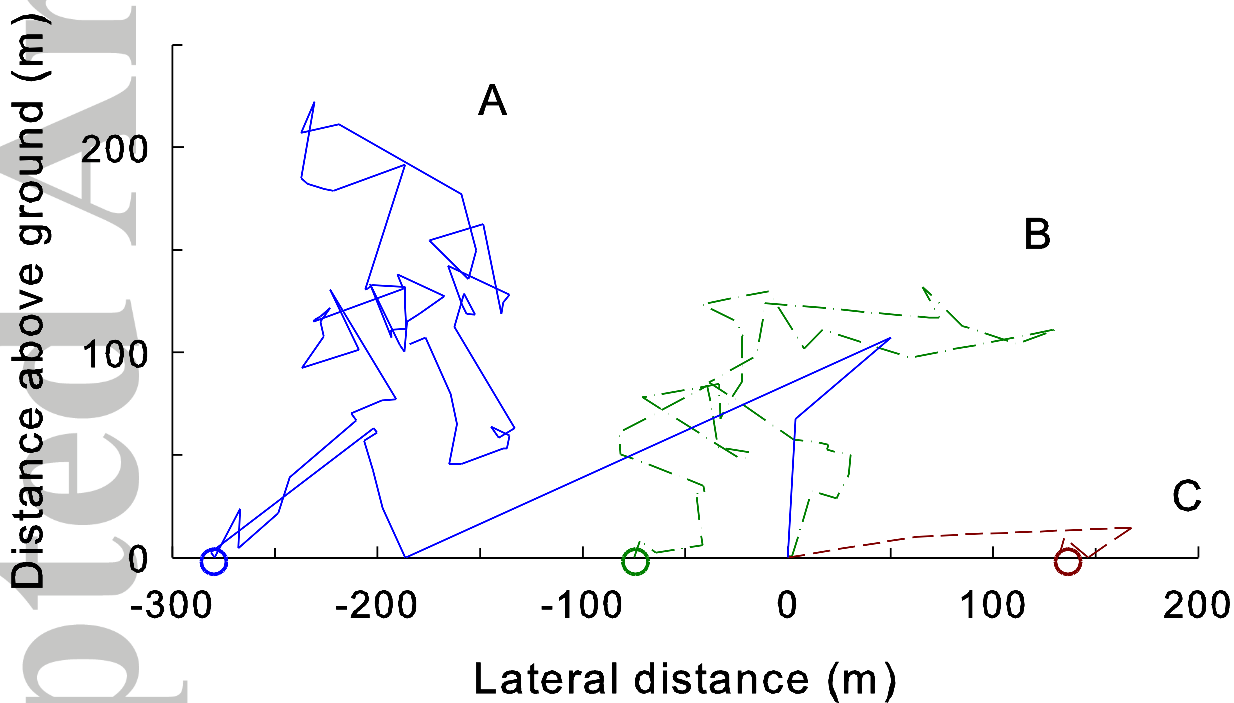


Figure 3. Three MCNPX random walks for particles emitted from the ground surface and detected within 2 meters of the ground. Particles emitted from the ground at *x*,*y*,*z*=0,0,0.

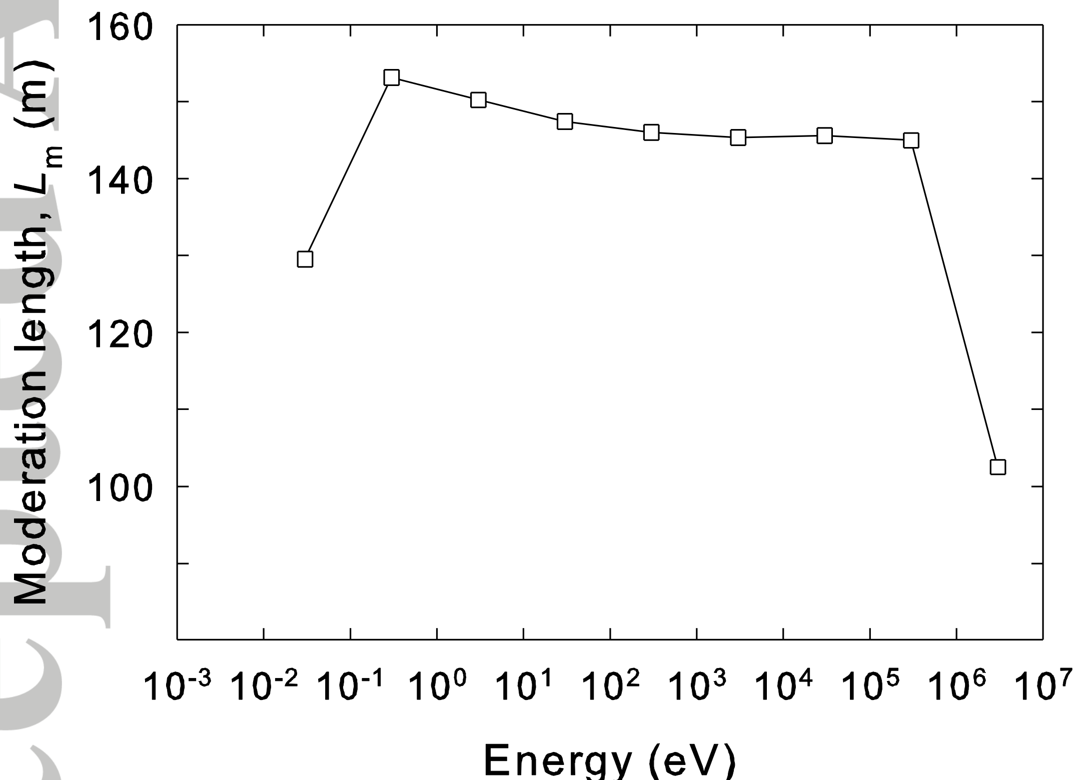


Figure 4. Moderation length, L_m, as a function of detection energy for a detector 0-2 m from the ground.

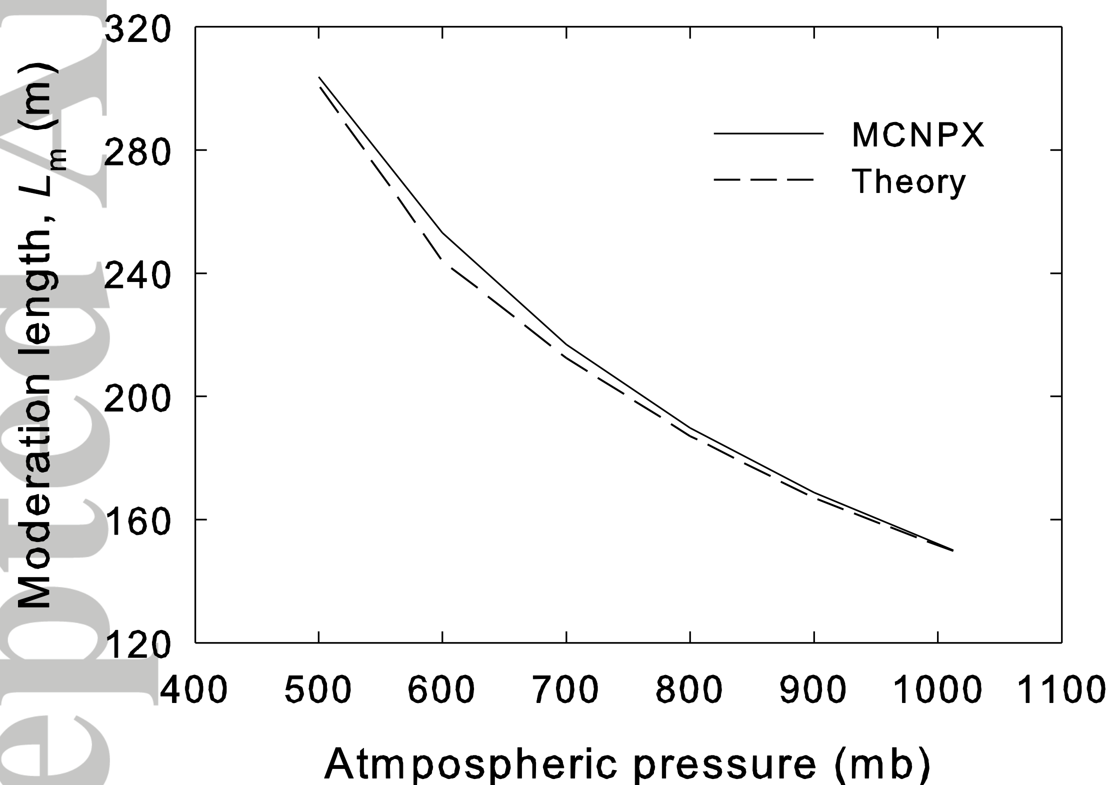


Figure 5. Moderation length as a function of atmospheric pressure from theory and MCNPX simulations.

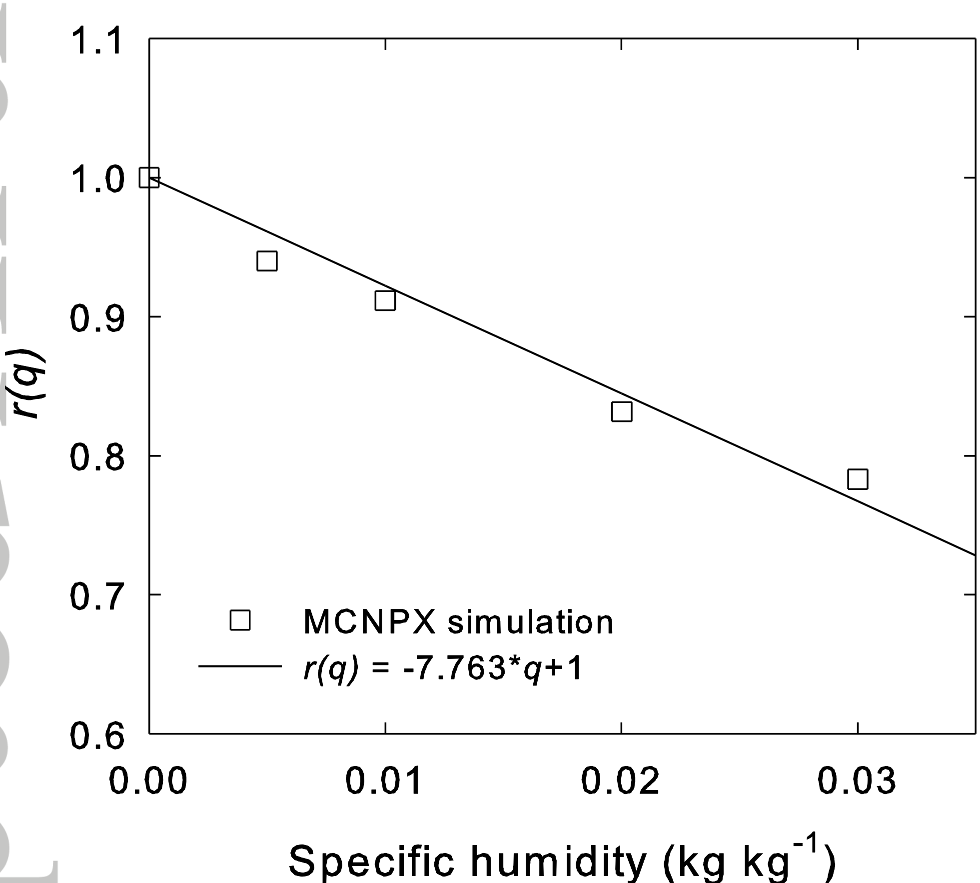


Figure 6. Dependence of the moderation length on the specific humidity of the atmosphere. Here $r(q)$ is the ratio of the moderation length in humid air, $L_m(q)$, to the moderation length in dry air length, $L_m(q=0)$.

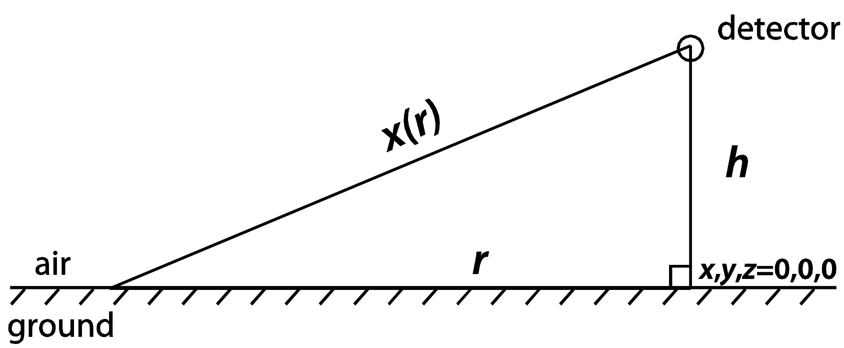


Figure 7. Geometric relationship between h, r and $x(r)$.

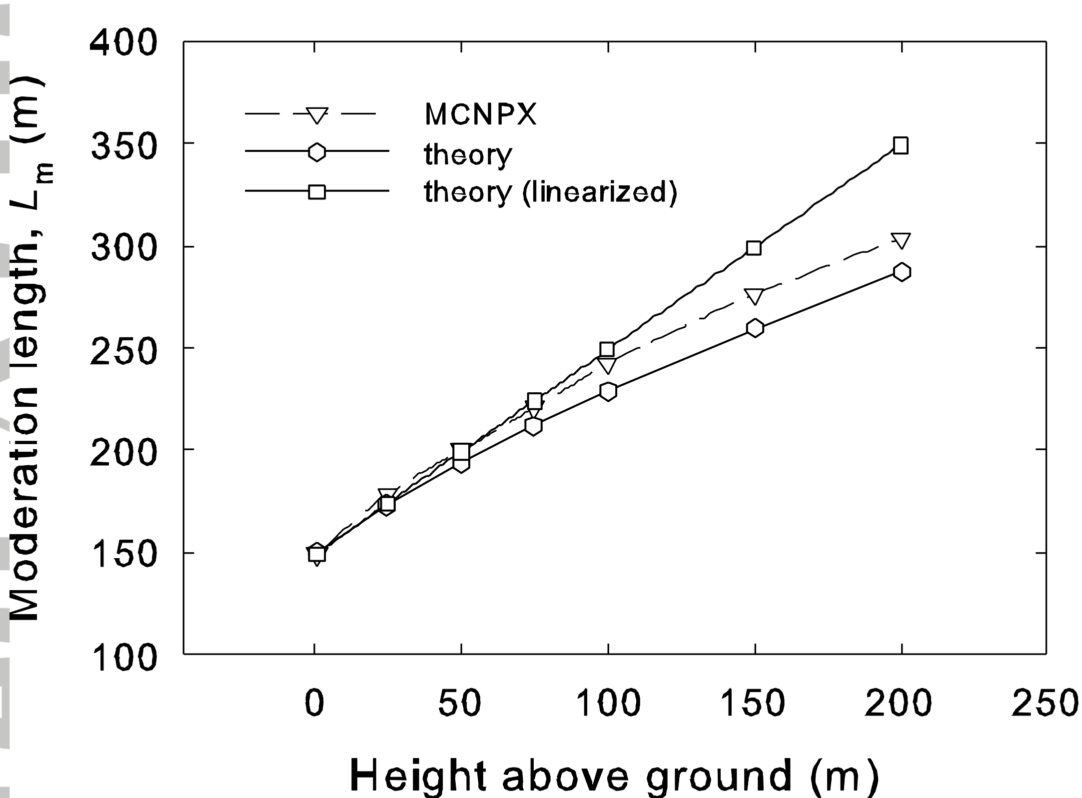
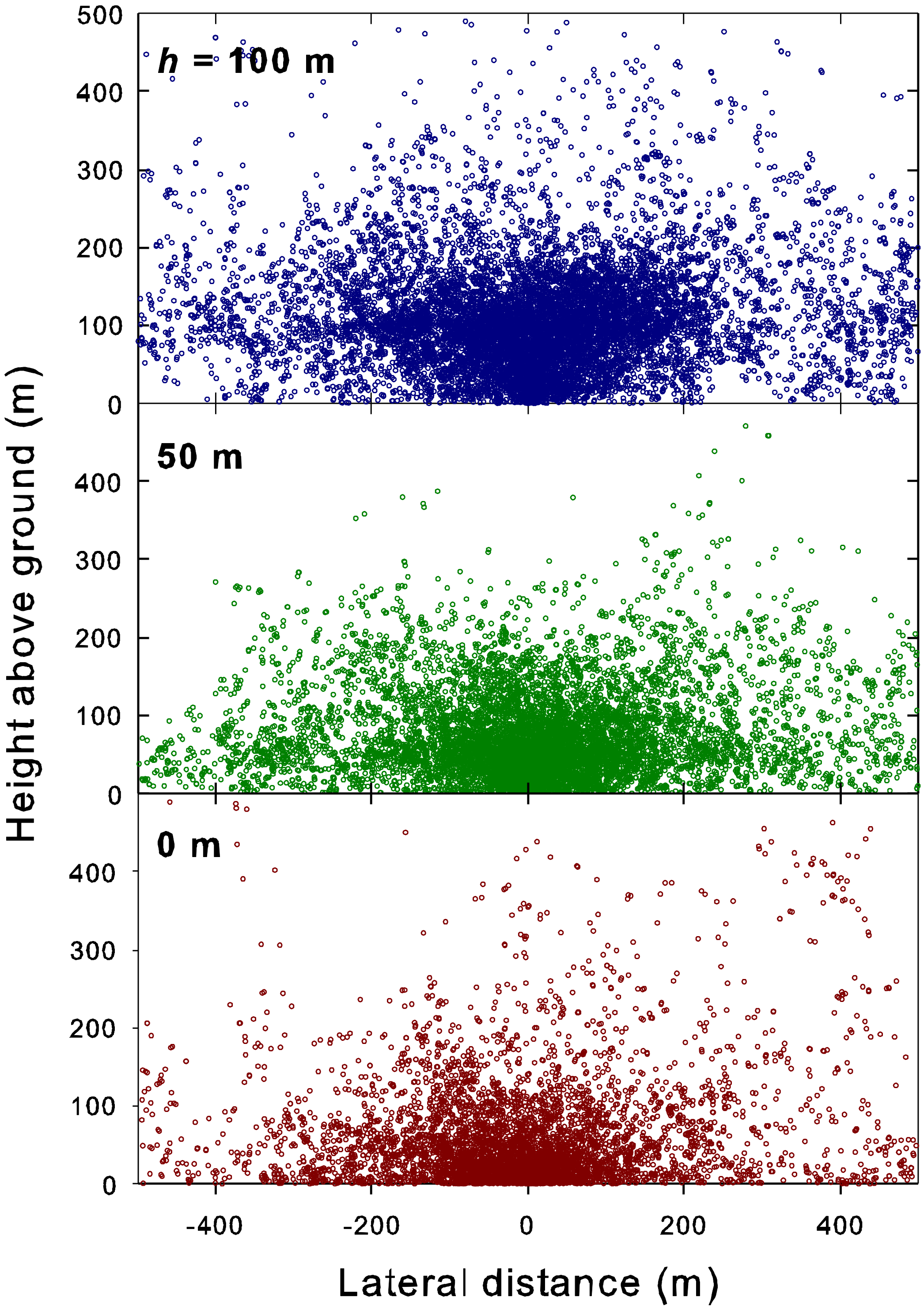


Figure 8. Moderation length as a function of height according to theory (Equations 22 and 23) and MCNPX simulations.



h = 100 m
50 m
0 m
Height above ground (m)
Lateral distance (m)
500
400
300
200
100
0
-400
-200
0
200
400

Figure 9. Atmospheric collisions for particles emitted from x,y,z=0,0,0 and recorded in a detector layer placed at 0 m, 50 m, and 100 m above the ground. Each plot shows 20,000 collisions.