Footprint characteristics revised for field-scale soil moisture monitoring with cosmic-ray neutrons

M. Köhli¹, M. Schrön², M. Zreda³, U. Schmidt¹, P. Dietrich², and S. Zacharias²

¹Physics Institute, Heidelberg University, Heidelberg, Germany, ²Department of Monitoring and Exploration Technologies, UFZ—Helmholtz Centre for Environmental Research, Leipzig, Germany, ³Department of Hydrology and Water Resources, University of Arizona, Tucson, Arizona, USA

Abstract Cosmic-ray neutron probes are widely used to monitor environmental water content near the surface. The method averages over tens of hectares and is unrivaled in serving representative data for agriculture and hydrological models at the hectometer scale. Recent experiments, however, indicate that the sensor response to environmental heterogeneity is not fully understood. Knowledge of the support volume is a prerequisite for the proper interpretation and validation of hydrogeophysical data. In a previous study, several physical simplifications have been introduced into a neutron transport model in order to derive the characteristics of the cosmic-ray probe’s footprint. We utilize a refined source and energy spectrum for cosmic-ray neutrons and simulate their response to a variety of environmental conditions. Results indicate that the method is particularly sensitive to soil moisture in the first tens of meters around the probe, whereas the radial weights are changing dynamically with ambient water. The footprint radius ranges from 130 to 240 m depending on air humidity, soil moisture, and vegetation. The moisture-dependent penetration depth of 15 to 83 cm decreases exponentially with distance to the sensor. However, the footprint circle remains almost isotropic in complex terrain with nearby rivers, roads or hill slopes. Our findings suggest that a dynamically weighted average of point measurements is essential for accurate calibration and validation. The new insights will have important impact on signal interpretation, sensor installation, data interpolation from mobile surveys, and the choice of appropriate resolutions for data assimilation into hydrological models.

1. Introduction

Whenever hydrology, agriculture, or climate change is concerned, the endeavor to find efficient methods of quantifying water resources is vitally important. Extensive monitoring of soil moisture and above-ground water storage is of key importance to constrain hydrological model predictions or to control management systems for irrigation. However, small-scale variability of soil moisture has always been an issue for the interpretation and application of point measurements [Vereecken et al., 2008; Biswas, 2014]. At large scales, remote sensing methods provide near-surface estimates of soil moisture. However, drawbacks are shallow penetration depth, low temporal resolution, and significant influence of surface conditions [e.g., Wagner et al., 2007]. From the modeler’s perspective, information at scales other than the modeling scale requires procedures for rescaling which introduce uncertainty during the assimilation process [Vereecken et al., 2007].

The method of cosmic-ray neutron sensing (CRNS) [Zreda et al., 2008, 2012] has proven to be effective in serving representative data at relevant scales. The reported footprint radius of \( \approx 300 \) m [Desilets and Zreda, 2013] is much larger than spatial correlation lengths of soil moisture patterns, typically ranging between 30 and 60 m [Western et al., 2004]. Thereby, this technology outperforms conventional in situ measurements in terms of representativeness for scales beyond several tens of meters.

Neutron radiation is omnipresent in the atmosphere as it is generated by a nearly constant incoming flux of cosmic rays. The presence of hydrogen near or in the ground reduces the neutron abundance in a predictable way. Especially, the density of fast neutrons in air can serve as an efficient proxy for the quantity of ambient water. The continuous monitoring of background radiation is a passive and noninvasive solution to the problem of representativeness, because the integral average over local water sources is an intrinsic property of the method.
Generally, spatial integration in the support volume of a measurement is intrinsic to most instruments in hydrogeophysics. When it comes to interpretation and validation, however, accurate knowledge of the spatial extent and sensitivity to environmental conditions is indispensable. For that reason, investigating an instrument’s support volume is an active field of research in hydrogeophysics, e.g., considering time-domain reflectometry [Ferré et al., 1996, 1998], ground-penetrating radar (see Huisman et al. [2003] for a review), and nuclear magnetic resonance [Legchenko et al., 2002; Lubczynski and Roy, 2004] among others. Methods based on the global positioning system [Larson et al., 2008] or gravimetry [Creutzfeldt et al., 2010; Kazama and Okubo, 2009] exhibit footprints comparable to the cosmic-ray neutron probe. However, the exact spatial sensitivity often remains unclear and thus limits the interpretation of measurements. In planetary space science, investigating an instrument’s footprint is of fundamental importance, for instance, to improve the resolution and interpretation of gamma or neutron measurements [Lawrence et al., 2003; Maurice et al., 2004]. Monte Carlo simulations were consulted to inquire the geophysical support volume literally in depth [McKinney et al., 2006].

Using neutron transport modeling based on the Monte Carlo method, footprint characteristics of the CRNS technique were presented initially by Zreda et al. [2008] and investigated in detail by Desilets and Zreda [2013] for idealized environmental conditions. The latter laid an important foundation to plan and improve sampling strategies and local site arrangement. According to Zreda et al. [2012], coastal transect experiments confirmed the reported footprint radius of several hundreds of meters, but the detailed interpretation of these measurements appears to be challenging. Recent investigations with mobile neutron detectors suggest that the sensor responds to remote water bodies below the accepted theoretical distance. Furthermore, during the course of preliminary investigations the authors could observe an effect of extraordinary sensitivity to the very first meters around the sensor. Thus, doubts about the accepted exponential decrease are raised by measuring close to a shoreline or a small group of people, or comparing signals of many colocated sensors in a small patch. By using an alternative neutron source in the simulation, Rosolem et al. [2013] found that the detector is sensitive to water vapor in heights above the probe ranging from 412 to 265 m for dry and wet air, respectively. Their results indicate that the assumptions on the modeled neutron source are decisive.

A more detailed understanding of the sensor’s support volume becomes important as research projects expand to complicated terrain and mobile applications [e.g., Dong et al., 2014]. An increasing number of CRNS probes are covering heterogeneous land which is often partly equipped with soil moisture monitoring networks [e.g., Han et al., 2014; Hawdon et al., 2014; Zhu et al., 2015]. Previous studies focused on the applicability and evaluation of the CRNS method, where spatiotemporal conditions have mostly been homogeneous [e.g., Franz et al., 2012a]. Thereby it was difficult to identify invalid assumptions on the spatial sensitivity when point measurements were averaged. Bogena et al. [2013] identified this problem and applied a horizontally weighted average based on simulations from Zreda et al. [2008]. But the authors did not compare it to the nonweighted average and so the open question remains whether this approach is advantageous. Coopersmith et al. [2014] introduced Voronoi-weights to differently vegetated parts of the footprint. However, the distance to the cosmic-ray neutron sensor was not accounted for. In all these cases, a proper spatial weighting concept based on distance, depth, and environmental conditions may lead to improved matching between the cosmic-ray derived soil moisture and the averaged validation data. Moreover, Franz et al. [2013b] show that large heterogeneous structures in the footprint can affect the average soil moisture signal apparent to sensors, because the neutron density and water content are nonlinearly related [Desilets et al., 2010]. This phenomenon again indicates that a proper spatial weighting of dry and wet spots could help to compensate for heterogeneity in the field.

To address the controversy about the footprint mentioned above as well as the needs for an accurate weighting function, we aim to minimize the number of physical simplifications in the numerical model. For example, many previous studies restrict simulations of neutron transport to very dry conditions in air or soil [e.g., Zreda et al., 2012; Franz et al., 2013b; Zweck et al., 2013] and thus neglect the enormous influence of even small hydrogen sources to the fate of neutrons. Moreover, we are paying particular attention to a proper choice of the neutron source as model input.

In the past years, various types of neutron source models have been chosen for particle transport simulations in order to study local effects of cosmic-ray neutron interactions. A common approach is to mimic incoming galactic cosmic rays by locating a neutron source at ≈ 8 km altitude and by sampling the neutron
energies randomly from a primary cosmic-ray spectrum in a regime much above 100 MeV [e.g., Franz et al., 2013b; Rosolem et al., 2013; Zweck et al., 2013]. On the other hand, Desilets and Zreda [2013] and Shuttleworth et al. [2013] applied an artificially distributed source below ground and sampled from an evaporation spectrum that peaks at \( \approx 1 \text{ MeV} \). This approach makes the consequential assumption of a suppositional incoming cosmic-ray spectrum that is comprised of high-energy neutrons only. We will discuss both strategies in section 2.5. The short review shows that there is no clear agreement about proper energies and the location of the source. But as Glasstone and Edlund [1952] and Desilets and Zreda [2013] argue, neutron transport is highly sensitive to the neutron’s initial energy.

Elaborated studies about atmospheric particle transport led to important progress in finding a reliable energy spectrum for cosmic-ray neutrons. Sato and Niita [2006] and Sato et al. [2008] simulated cosmic rays in the atmosphere covering a wide range of altitudes, cutoff-rigidities (roughly correlated to latitudes) and solar modulation potential. Analytical descriptions for neutron energy spectra were provided that have been validated with independent measurements. By choosing a parameterized energy spectrum of this kind, Lifton et al. [2014] recently resolved some long lasting discrepancies among scaling models for cosmogenic nuclide production.

In the same manner, we utilize the full available energy spectrum from Sato and Niita [2006] near the ground to refine previous neutron transport calculations. The objective of this study is to specify the footprint volume and radial sensitivity for various environmental conditions from arid to humid climate. We further investigate some of the open questions regarding the influence of topography, terrain, and water content in soil, air, and vegetation.

2. Theory

Recognizing the complexity of environmental neutron physics is essential to interpreting observations and simulations. The neutron’s sensitivity to specific types of atoms can be high, but it further depends on neutron energy which in turn decreases with every interaction [Rinard, 1991]. For instance, even small changes in the abundance of hydrogen can dramatically alter neutron interactions. Model simplifications, while necessary, should be introduced only with great care and their possible effects need to be assessed.

2.1. Primary Cosmic Rays

Cosmic radiation that is pounding the Earth originates mostly in our galaxy, e.g., from acceleration in shock regions of supernova remnants (see Blasi [2014] for a review). Protons are the main part of the particle flux, accompanied by other charged nuclei. The energy spectrum of the primary cosmic rays peaks at around 1 GeV per nucleon. Depending on their momentum, cosmic-ray particles may pass the geomagnetic fields...
of the Sun and the Earth. The solar magnetosphere leads to temporal variations of the cosmic-ray intensity based on the solar activity index. The planetary magnetosphere prevents cosmic rays from entering the atmosphere by deflecting particles below an energy-dependent cutoff rigidity $r_c$. Both effects further decrease the typical energies of incoming radiation to several hundreds of MeV per nucleon [e.g., Nesterenok, 2013; Grimani et al., 2011].

2.2. Neutron Generation in the Atmosphere

Secondary cosmic-ray particles (e.g., muons, protons, pions, neutrons) are generated by electromagnetic and nuclear interactions mostly in the outer part of the Earth’s atmosphere. Their intensity peaks at the Pfotzer maximum (50–100 g/cm$^2$ atmospheric depth [Pfotzer, 1936]) and decreases exponentially by several orders of magnitude toward sea level. However, altitudinal effects on the shape of the energy spectrum appear to be marginal [e.g., Nesterenok, 2013; Sato and Niita, 2006; Hands et al., 2009; Kowatari et al., 2005; Lei et al., 2005]. Typically, high-energy protons induce spallation of nitrogen or oxygen nuclei in the atmosphere [e.g., Letaw and Normand, 1991]. This reaction releases a couple of neutrons which are in turn able to trigger further cascades. The physics of high-energy neutron interactions is not well-known and thus the attempt to describe the complete process is accompanied by uncertainty.

2.3. Energy Reduction by Air, Soil, and Water

Above thermal energies, neutrons lose energy with every collision and cannot accelerate to higher energies due to their neutral electrical charge. The final energy spectrum for neutrons at ground level is depicted in Figure 1, where three peaks are prominent. Highly energetic neutrons at $\approx 100$ MeV (red) are produced by intranuclear cascades and preequilibrium processes [Gudima et al., 1983]. When high-energy neutrons or protons interact with air or soil, the excited nuclei evaporate (i.e., release) the so-called fast neutrons at a lower energy. This process manifests itself at the peak at $\approx 1$ MeV (green) and shows additional absorption fine structure due to distinct resonances of nonhydrogen atoms. Neutron interactions in the sub-MeV region (blue) are dominated by elastic collision, in which, as a rule of thumb, the energy loss is correlated to the mass of the target nucleus. Due to the extraordinarily low mass of hydrogen, this energy band is most sensitive to water and organic molecules and thus most relevant for the CRNS method. Below $\approx 1$ eV, the target is usually in thermal equilibrium with the environment. Here the target’s energy significantly contributes to the neutron’s energy when transferred during a collision. As a consequence, neutrons finally become thermalized at $k_B T \approx 25$ meV (grey), where $k_B$ is the Boltzmann constant. Since neutrons cannot leave the thermal equilibrium toward relevant energies, this work disclaims the lower part of the spectrum.

The probability of a neutron to interact with an atom is quantified by the term cross section. It exhibits large variations due to isotopic composition and atomic number. The atomic mass $A$ classifies types of possible interactions. Very heavy nuclei with $A > 80$ like $^{235}$U, can undergo fission and scatter fast neutrons inelastically. Intermediate nuclei of $25 < A < 80$ are able to absorb or scatter the neutron inelastically. Light nuclei with $A < 25$ predominantly perform elastic scattering of neutrons. The hydrogen nucleus, $A = 1$, exhibits exceptionally high absorption and elastic scattering cross sections. However, absorption is only significant in the thermal energy regime to which the ideal detector for measuring soil moisture is insensitive. Figure 2 shows neutron elastic scattering cross sections for elements that are most abundant in terrestrial air, water,
and solids. Water vapor, oxygen, and nitrogen are particularly responsible for the neutron’s deceleration in air. Therefore, the range a neutron can travel before thermalization is expected to increase with altitude (i.e., decreasing air density) and decrease with increasing air humidity. On the other hand, dense soils, organic matter, or soil water content are expected to reduce the penetration depth.

2.4. Spatial Mixing and Its Analytical Description

Evaporated neutrons perform a random walk type of propagation, because the angular probability distribution of their interactions is mostly isotropic. In contrast, incoming high-energy neutrons do preferably interact in a forward directed process and thus vanish into the soil without reflection. In air, fast neutrons typically travel several tens of meters between collisions. And since hydrogen is most efficient in absorbing energy from the neutron, whereas air is less, information about remote water bodies can quickly propagate within hectometers. This process leads to a nearly homogeneous neutron density in the air which can be sampled locally by the cosmic-ray neutron sensor and represents an average of ambient hydrogen abundance in soil, air, and vegetation.

As a first order approach, one could expect neutrons to behave as a diffusive gas, as it was formulated by Glasstone and Edlund [1952], and applied to a footprint estimate by Desilets and Zreda [2013] besides the modeling. But since every collision with a particle results in an energy loss for the neutrons, their mean free path between collisions changes and diffusion theory loses validity. The Fermi age theory (e.g., applied in Barkov et al. [1957]) accounts for these energy losses in a diffusive system, but analytical solutions exist only for monoenergetic particles and are not feasible for the cosmic-ray neutron spectrum exposed to a wide range of environmental atoms with different cross sections.

For these reasons, the assumption of homogeneous diffusion as applied by Desilets and Zreda [2013] will bias results toward neutrons with higher energies. Furthermore, the single-layer diffusion approach neglects the influence of the soil. Since neutrons interact with the soil and its water content on their path to the sensor, their energy is reduced more efficiently compared to the propagation in air. Therefore, the homogeneous analytical approach overestimates the horizontal footprint radius and is rather valid for a vertical footprint above the surface. The phenomenon of exceptionally high vertical footprints was shown experimentally with a detector on a helicopter by M. Zreda (unpublished data, 2013) and indirectly with numerical simulations [Rosolem et al., 2013].

It is not feasible to join the complex problem of neutron transport, multienergetic Fermi age theory, and two-layer diffusion theory into a deterministic solution. Therefore, statistical and numerical approaches are the only way to include all necessary factors involved.

2.5. In Quest of a Proper Model Input

Simulations of cosmic-ray neutrons near the ground require consolidated knowledge of the incoming radiation. However, the location of the source is commonly traded against computational effort, whereas the initial energy spectrum is bonded to a variety of uncertainties.

A popular approach is to launch secondary cosmic-ray neutrons at ≈ 8 km altitude and to perform their propagation through the atmosphere [e.g., Franz et al., 2013b; Rosolem et al., 2013; Zweck et al., 2013]. This strategy and related simplifications come with several drawbacks:

1. Cross sections of high-energy neutrons exhibit uncertainties of up to 50% depending on element and type of reaction, though there has been progress in the last two decades [e.g., Salvatores et al., 1994; Palmiotti et al., 2007]. As a consequence, inconsistencies are apparent throughout different codes for galactic and atmospheric cosmic-ray transport [e.g., Lin et al., 2012; Lei et al., 2005; Sheu and Jiang, 2003].
2. Measurements of cosmic-ray energy spectra are additionally accompanied by observational uncertainties. Comparative studies of Monte Carlo codes show differences of up to 20% for calculating sensitivities of the neutron response to experimental devices [Barros et al., 2014] and as well for the spectrum unfolding technique [Rühl et al., 2014].
3. The exclusive neutron source at the top of the modeled atmosphere inadvertently neglects neutron generation throughout the atmosphere by other secondary particles like protons, pions, and muons.
4. Atmospheric water vapor is often ignored, although hydrogen is the main moderator for neutrons.
5. The large difference in scale of the domain requires high computational effort to reach sufficient statistics.
Models which rely on particle propagation through the upper atmosphere incorporate a high complexity and vulnerability to such uncertainties involved.

In the attempt to reduce computational effort, other studies identified the high-energy component of the cosmic-ray neutron spectrum as the precursor for the generation of fast neutrons in the soil [Zreda et al., 2008; Desilets and Zreda, 2013; Shuttleworth et al., 2013]. Since the attenuation process of high-energy neutrons in the ground is known, it seems likely that an artificial source in the soil is sufficient to mimic the evaporative production of relevant neutrons. However, some drawbacks of this method are important to note:

1. Attenuation of high-energy neutrons in the soil follows an exponential decrease that is dependent on soil type and location on Earth [Gosse and Phillips, 2001].

2. There is no verified energy spectrum for neutrons in the soil. Evaporation neutrons are a significant part, but do not make up the spectrum as a whole (see Figure 1).

3. In reality, the incoming energy spectrum from the atmosphere exhibits low-energy components and particularly neutrons which already evaporated in the air. On entering the soil, at least one interaction is needed to alter the direction back to the surface. In contrast, the artificially generated neutrons in the soil can escape without any interaction.

Considering only the evaporative neutrons in the soil can be a decent approach, especially for dry conditions. However, this strategy tends to overestimate average neutron energies, as incident low-energy neutrons from the top are neglected, and thus also overestimates the footprint size. Moreover, the deduced footprint appears to be insensitive to soil moisture, because its influence on neutron moderation is underestimated.

In this work, a different approach is applied, which aims to combine the advantages as well as avoid the drawbacks of both strategies mentioned above. To minimize the uncertainties of the propagated energy spectrum, this study focuses on the domain close to the surface by using validated results from independent atmospheric simulations as model input. This concept is computationally efficient and represents an established approach in planetary space science [e.g., Tate et al., 2013].

Cosmic-ray propagation in the atmosphere has been modeled thoroughly by Sato and Niita [2006]. They provide a reliable energy spectrum of cosmic-ray neutrons for a variety of altitudes, cutoff-rigidities, solar modulation potential, and surface conditions. These simulations have been validated with various independent measurements at different altitudes and locations on Earth. Moreover, the analytical formulations of the spectra turned out to be effective in use for subsequent calculations. The presented energy-dependent flux $\phi(E)$ is described by a mean basic spectrum $\phi_b$, a function for neutrons below 15 MeV $\phi_{1}$, an extension for thermal neutrons (disclaimed in this work), and a modifier $f_{\text{sp}}$ for the geometry of the interface:

$$\phi(s, r_c, d, E, w) = \phi_b(s, r_c, d, E) \cdot f_{\text{sp}}(E, w) \cdot \phi_1(s, r_c, d),$$

where $w$ is the weight fraction of water in the ground. We focus our study on parameters for the atmospheric depth near sea level, $d = 1020$ g/cm$^2$, solar maximum conditions $s = 1700$ MV and an exemplary cut-off rigidity of $r_c = 10$ GV. This selection might introduce small differences for different places on Earth. However, Goldhagen et al. [2004] show that geomagnetic latitude has only very small effects on the shape of the spectrum. It depends slightly on atmospheric depth, as discussed by Sato and Niita [2006] and found by various authors [e.g., Sheu and Jiang, 2003]. However, this is only significant for altitudes above several kilometers (see also section 2.2).

As such spectra generally consist of an incoming as well as a backscattered component, the appropriate incident spectrum was separated as follows. First, for the given spectrum the response spectrum is calculated over pure water ($w = 1$), where the incoming component is dominant and thus the uncertainties of the calculation are minimal. Tracing the neutrons allows to determine an energy-dependent multiplicity function $m(E)$ which allows to separate incoming ($m = 1$) from scattered parts ($m > 1$) of the spectrum. This filter can now be used to “subtract” only backscattered neutrons from the original spectrum. A thus recalculated spectrum contains only incident neutrons and can be used as the source of incoming radiation for any surface condition. It is provided in the supporting information.
As an exception to the otherwise isotropic distribution, emission angles of high-energy neutrons above 10 MeV are highly collimated along the downward facing direction (nadir angle $\alpha$). According to observations and simulations by Nesterenok [2013], the nonuniformity of the angular spectrum $J(\alpha)$ is given by:

$$J(\alpha) = e^{-2 \cdot (1 - \cos \alpha)}.$$ 

The presented strategy combines a universal and validated input spectrum and angular distribution for cosmic-ray neutrons with a reduced number of simplifications and a high computational efficiency.

### 2.6. Footprint Definition

The footprint of a geophysical instrument generally covers the area in which the medium of interest is probed and the carrier of such information is detected. The scenario of a centrally located sensor which detects neutrons isotropically exhibits point symmetry and thus leads to the assumption of a circular footprint area, $A = \pi r^2$. In this work, we define the travel distance $r$ as the Euclidean distance between the point of detection and the point of the neutron’s first contact with the ground, also denoted as origin. Since $r$ depends on the neutron’s initial energy and number of collisions, it can range between 0 and $10^3$ m. Thus, a quantile definition is needed to find a definite distance $R$ within which most of the detected neutrons have probed the ground.

By assuming an exponential decay of detected neutron intensity over travel distance, which relates to the solution of a simple diffusion model, Zreda et al. [2008] and Desilets and Zreda [2013] legitimate the use of two $e$-folding lengths, i.e., the 86% quantile, in order to define the footprint radius. Figure 3 illustrates the radial decrease of the detected neutron intensity $W_r$ as a result of Monte Carlo simulations performed in this work. Although the calculated response does not exhibit a simple exponential shape, any other quantile would be an arbitrary choice as well. Careful interpretation of this value is recommended, however, because a high quantile value will always treat long-range neutrons with favor, regardless of how often they have probed the soil. Nevertheless, we decide to follow the definition of the 86% quantile for historical reasons and denote the according footprint radius with $R_{86}$ and the footprint area as $A = \pi R_{86}^2$.

![Figure 3. Detected neutron intensity dN/dr over distance r between origin and detection. The analytical fit is also called radial weighting function W_r, (left) Simulations were performed for humidity $h = 10$ g/m$^3$ and two exemplary soil moistures $\theta = 3\%$ and 50%. Quantiles declare footprint radii $R_{63}$ and $R_{86}$ (dotted) for $1 - e^{-1} \approx 63\%$ and $1 - e^{-2} \approx 86\%$ cumulative counts, respectively. Peaks at $r < 10$ m reach 57.5 for $\theta = 3\%$ and 37.9 for $\theta = 50\%$ according to the chosen scale. (right) Comparison of $W_r$ and the corresponding $R_{86}$ for four extreme cases of soil moisture and humidity. Both quantities are shaping the curves differently.](image-url)
The number of neutrons $N_R$ that have originated within a distance $R$ from the sensor is given by:

$$N_R = \int_0^R W_d \, dr.$$  \hspace{1cm} (1)

In order to find the distance within which 86% of the detected neutrons originate, the following equation is solved for $R_{86}$ numerically:

$$\int_0^{R_{86}} W_d \, dr = 0.86 \int_0^\infty W_d \, dr.$$  \hspace{1cm} (2)

In analogy, we define the penetration depth $D_{86}$ in the soil as the integral of a depth weighting function $W_d$ which is expected to also decrease with distance $r$ to the sensor.

3. Modeling

Monte Carlo simulations are able to track the histories of millions of neutrons. By taking all relevant physical interactions into account, the summary statistics of a large number of neutrons can reveal insights into their collective effects. In this study, we apply the Monte Carlo method to address both large geometric scaling and anisotropic conditions.

3.1. Software

To address the specific needs of neutron-only interactions, we developed the Ultra Rapid Adaptable Neutron-Only Simulation (URANOS) based on the Monte Carlo approach for neutron transport. The software was originally developed to simulate specific characteristics of the Heidelberg neutron spin echo detectors and was adapted to the cosmic-ray neutron problem. The physics model follows the implementation declared by the ENDF database standard and described by OpenMC [Romano and Forget, 2013], a recent Monte Carlo code alternative to MCNP. It features the treatment of elastic collisions in the thermal, epithermal, and fast regime, as well as inelastic collisions, absorption, and absorption-like processes (e.g., evaporation) which play a dominant role for the given elements (these are the processes described by the ENDF MT identifiers 5, 103, 107, 208, 209, and 210). Cross sections, energy distributions, and angular distributions were taken from the databases ENDF/B-VII.1 [Chadwick et al., 2011] and JENDL/HE-2007 [Shibata et al., 2011].

The URANOS code was tailored to the problem of neutron transport in environmental science. By neglecting unnecessary physical processes (e.g., fission and gamma cascades) this leads to a significant increase in the computing speed compared to other available Monte Carlo codes for the description of neutron transport. In preparatory studies, we explored the performance of the URANOS model in reproducing results from standard software like MCNPX. The tests successfully agreed in many different setups (not shown) such as the one presented by Sato and Niita [2006]. Particular attention was turned to the reproduction of the results from MCNPX performed by Desilets and Zreda [2013]. Using exactly the same setup of soil composition and source definition, we were able to reproduce the reported footprint radius of $\approx 300$ m and confirm the negligible dependence on soil moisture.

3.2. Neutron Source

Neutrons are launched from point sources randomly distributed in the region from 2 to 42 m above the surface (Figure 4). Energies are sampled from a precalculated spectrum based on Sato and Niita [2006], which is provided in the supporting information. High-energy neutrons are launched with a collimated angular distribution (see section 2.5). The source intensity was chosen according to statistical errors. More neutrons would lead to more accurate and smooth data. We experienced that $10^7$ neutrons are a reasonable trade-off between computational effort and precision.

3.3. Detector

Neutrons are recorded individually in an horizontally infinite detector layer. Any neutron that experienced interaction with the soil is counted as it passes the layer. The infinite plane detector overlays the atmosphere by means of a 25 cm high sheet at a vertical position of 175–200 cm. This geometry was chosen because we aim to compare our results with Desilets and Zreda [2013], who tallied the neutron fluxes in a 2 m detector layer. The detector layer is crossed by the neutrons and thus maps the spatial field of neutron densities. It is
an appropriate abstraction of a realistic, small-scale detector volume of the same height that absorbs neutrons. As tests confirm, multiple counts of a single neutron in the detector layer account for the measured density equivalent for a single count per volume detector. This relation holds if (1) the dimension of the absorbing detector medium stays below typical scale lengths of neutron interactions (10–100 m), and (2) particles do not scatter multiple times in that volume. That is very unlikely for non-thermal neutrons and furthermore does not factorize in the count statistics.

We refer to several statements of the effective energy range to which the detector is sensitive. Following practical considerations by Desilets and Zreda [2013] and theoretical by Hertel and Davidson [1985], the detection energy is set to a window from 10 to $10^3$ eV. Kouzes et al. [2008] reports that the detection efficiency of moderated helium-3 detectors is nearly constant in that energy regime, which is why signal weighting for different energies is not needed.

Recent studies reported that the common cosmic-ray neutron detectors (presented by Zreda et al. [2012]) are contaminated by $\approx 30\%$ thermal neutrons [McJannet et al., 2014]. We do not account for this issue, because this study aims to investigate characteristics for a detector ideally tailored to the needs of environmental water sensing.

### 3.4. Air, Soil, and Water

The modeled pure air medium consists of $78\%_{\text{vol}}$ nitrogen, $21\%_{\text{vol}}$ oxygen, and $1\%_{\text{vol}}$ argon at a pressure of 1020 mbar. The soil extends to a depth of 6 m and the air to 1000 m. Both, soil and air are represented by planes of infinite extension, which can have subdomains, either to create a density profile in depth or to add specific entities like water or a detector. The soil consists of $50\%_{\text{vol}}$ solids and a scalable amount of $H_2O$. The solid domain is comprised of $75\%_{\text{vol}}$ SiO$_2$ and $25\%_{\text{vol}}$ Al$_2$O$_3$ at a compound density of 2.86 g/cm$^3$. Thus, the total densities vary from 1.43 g/cm$^3$ to 1.93 g/cm$^3$ for $0\%_{\text{vol}}$ and $50\%_{\text{vol}}$ soil moisture, respectively.

Further chemical constituents regarding rock types are not significant for fast neutron moderation, according to calculations from Zreda et al. [2012] and Franz et al. [2012b] and the discussion in section 2.3.

### 4. Results and Discussion

The response of the ground to the incoming flux of cosmic-ray neutrons leads to several interesting features in the resulting energy spectrum. Figures 5a and 5b confirm the efficient reduction of neutron intensity by soil moisture in the relevant energy range of the CRNS method. Figure 5c shows that water vapor particularly affects neutrons at the upper end of the energies considered. In this energy domain, neutrons cover the largest distances and are consequently exposed to the highest path-integrated amount of air. In general, neutrons appear to be very sensitive to small amounts of hydrogen in soil and air.

#### 4.1. Radial Footprint Changes With Humidity and Soil Moisture

We performed simulations with a variety of volumetric water contents in the soil from $0\%$ to $50\%$ and absolute humidity in the air from 0 to $50\, \text{g/m}^3$. Figure 3 illustrates the sensitivity of the detector to neutrons originating at different radial distances $r$. This radial weighting function $W$, can assist in finding a properly weighted mean of independent soil moisture measurements. It further shows that little contribution is made by neutrons from $r > 200\, \text{m}$ and highest contribution comes from $r < 10\, \text{m}$ around the sensor.
The peak at \( r < 10 \text{ m} \) accounts for neutrons that directly emerge from the ground and have a high probability to be detected even though most of them come from the lower part of the neutron energy spectrum. The region up to \( r < 50 \text{ m} \) describes the average mean free path of most of the environmental neutrons in humid air. For distances between 50 and 200 m, neutrons interact with the soil multiple times until they are detected, which in turn means that with increasing \( r \), average neutron energies quickly become insufficient in order to arrive at the detector before thermalization. From about 200 m on, detected neutrons are dominated by the higher energetic part of the spectrum, which appear to be higher in flux rates and are able to probe the soil very far from the detector.

Due to the different neutron energies involved, we found an accurate fit to the intensity distribution (Figure 3) by splitting the radial domain into four exponential parts. An analytical description can be obtained for \( \theta \geq 2 \% \):

\[
W_r(h, \theta) \approx \begin{cases} 
F_1 e^{-r/F_1} + F_2 e^{-r/F_2}, & 0.5 \text{ m} < r \leq 50 \text{ m} \\
F_3 e^{-r/F_3} + F_4 e^{-r/F_4}, & 50 \text{ m} < r < 600 \text{ m}
\end{cases}
\]  

(3)

where the parameter functions \( F_i(h, \theta) \) are individually dependent on humidity and soil moisture as given in Appendix A. The separation at \( r=50 \text{ m} \) accounts for the nontrivial shape of the function as described above. For \( r > 50 \text{ m} \) both exponential terms describe diffusion-like processes each accounting for soil moisture and air humidity presence. On the contrary, in \( r \leq 50 \text{ m} \) diffusion is not the main process, however, since the same functional structure still holds numerically, it was chosen for convenience. Following equation (2), we integrated \( W_r(h, \theta) \) numerically. The resulting \( R_{66}(h, \theta) \) is analytically difficult to grasp, thus we illustrate the numerically integrated results as contours in Figure 6 and present a numerical matrix in the supporting information. The contour plot shows that the footprint radius ranges from 240 to 130 m between arid and tropical climate, respectively.

The response to soil moisture variations is significant for humid climate between 10\%_{o_d} and 40\%_{o_d} as well as for very dry conditions <3\%_{o_d}. Previous studies underestimated the role of soil moisture for the footprint due to the choice of a modeled neutron source below the surface (see section 2.5). Comparative studies (not shown) indicated that this detail is the major cause for the discrepancy to findings from Desilets and Zreda [2013]. Moreover, the decrease of the footprint with increasing soil moisture does not necessarily imply that the area-average estimate is less representative. According to Korres et al. [2015], spatial variability of soil moisture tends to be low for rather wet soils. In this context, the effective representativeness of the CRNS method appears to be almost unchanged.

The response to variations of absolute humidity features a 10 m decrease of the footprint radius for every change of 4–6 g/m³ water vapor. Zreda et al. [2012] refers to \( \approx 10\% \) reduction of the footprint from dry to saturated air, which can easily span \( \approx 25 \text{ g/m³} \). This change corresponds to a 20% change in footprint radius calculated with URANOS. However, Desilets and Zreda [2013] investigated the influence of humidity
in further detail and found a 10 m decrease for every change of \( \approx 6 \text{ g/m}^3 \) humidity from MCNPX simulations with dry soil. This value is consistent with results from URANOS, whereas the slightly higher gradient is a consequence of the different energy spectra used in the models.

The function \( W_r \) lays the basis for a refinement of the commonly applied sampling strategy. The accepted method equally weights point measurements from three distinct radii \([Zreda et al., 2012; Franz et al., 2012a]\) which correspond to an exponential weighting function (see section 2.4). In contrast, the present work shows that (1) the first tens of meters provide dominant contribution to the signal in a rather nonexponential relation, and (2) the shape of the weighting function changes temporally as it is affected by variable moisture conditions. It is therefore not possible to elaborate a universal sampling strategy. As a rule of thumb, we recommend to take more samples closer to the probe (e.g., 0–10 m) than was previously recommended. Subsequently, data should be weighted with \( W_r(h, \theta) \) in a postprocessing mode (see Appendix B).

### 4.2. Uncertainty Analysis

In the simulated system containing soil, atmosphere, and a detector, uncertainties propagate nonlinearly due to the variety of neutron interactions involved. As an indication of their total effect, we analyzed uncertainties of our calculations by means of the influence on the footprint radius \( R_{86} \).

Variations of cross sections by their standard deviation, given in the ENDF database, lead to changes of \( R_{86} \) by 4%, 3%, and 2% for \( \theta = 3\% \), 10\%, and 40\%, respectively. The effect of elastic scattering dominates the budget by approximately 70\%. Further details about this analysis are provided in the supporting information. The errors of the cross sections can be considered as systematic for neutron transport simulations in general. We further analyzed the impact of different source spectra as model input in a test case with 10% soil moisture and 5 g/m\(^3\) air humidity. As explained in section 2.5, the incident spectrum was generated over water by subtracting the soil response from the original mixed spectrum. Variations of this soil response spectrum by 20\% alter \( R_{86} \) by 2.5\%. If the emission angles of source neutrons were not set according to their angular distributions, but chosen perpendicular to the surface, the change of the footprint radius would be 2.5\% applied to high energetic neutrons only and 3.0\% using sub-MeV neutrons. Compared to the uncertainties involved in our calculations the impact of other source spectrum models can be much higher. The integration of the counted particles (equation (2)) further leads to statistical uncertainties on \( R_{86} \) in the order of 0.2\% for \( 10^7 \) neutrons.

All in all we conservatively report a total error of \( \Delta R_{86} = 4–6\% \), which scales from wet to dry conditions.
4.3. Footprint Scaling With Vegetation and Air Pressure

To investigate the footprint variability under vegetated conditions, we modeled above-ground vegetation as a layer of height $H_{\text{veg}}$, containing an exemplary mixture of water and carbon with a density of $\rho_{\text{veg}}=0.005$ g/cm$^3$. This corresponds to $\approx 4.4$ kg/m$^3$ biomass water equivalent (BWE), since we have chosen the molecular composition of the gas in a way that the living plant consists of $\approx 12\%$ carbon by weight. For layer heights below a few meters, variations of either density or height have comparable effect on neutron moderation. Therefore, this method can provide valid estimations of the vegetation effect in terms of both, height and water equivalent.

From the perspective of the neutron, the layer introduces a new source of hydrogen in the air and consequently reduces the traveling range in the same manner as humidity. For example, the footprint radius is reduced by $\approx 20\%$ for crops of height $H_{\text{veg}}=2$ m (BWE $\approx 8.8$ kg/m$^3$) in dry soils. From simulations presented in Figure 7a, we find an exponential dependence of the footprint scaling factor $f_{\text{veg}}$ on vegetation height $H_{\text{veg}}$:

$$f_{\text{veg}}(\theta) = 1 - 0.17 \left(1 - e^{-0.41H_{\text{veg}}^{\theta}}\right) \left(1 + e^{-7\theta}\right),$$

(4)

where $\theta$ is given in units of m$^3$/m$^3$. For thin vegetation cover, a linearization in $H_{\text{veg}}$ is appropriate. As Franz et al. [2013c] demonstrate, water in aboveground biomass influences the signal in another way than homogeneously distributed soil moisture, which is well reflected by the URANOS model approach (see also Figure 5).

On the other hand, the footprint can also expand with decreasing air pressure (e.g., increasing altitude). The lower air density allows neutrons to cover longer distances between collisions. For example, the footprint can be $20\%$ larger at a $\approx 2000$ m altitude ($\approx 800$ mbar) compared to sea level. Although a reciprocal fit is a reasonable estimate [Desilets and Zreda, 2013], our results presented in Figure 7b indicate an exponential dependence on $p$ due to the presence of hydrogen:

$$f_p = \frac{0.5}{0.86 - e^{p/p_r}} \approx \frac{p_0}{p}. $$

(5)

However, differences between the two models appear to be insignificant.

By taking the scaling factors into account, the final footprint radius can be estimated with:

$$R_{\text{fs}}(h, \theta, p, \text{veg}) = f_p \cdot f_{\text{veg}}(\theta) \cdot R_{\text{B}}(h, \theta).$$

(6)

An extreme case of vegetation is a forest site, where cosmic-ray sensors are placed to study the influence of wet biomass or interception water in the canopy [e.g., Desilets et al., 2010]. In order to provide a first glimpse of the influence of a forest on the footprint, we set up a gas representing the molecular composition of an exemplary forest with a density of $\rho_{\text{forest}}=0.0016$ g/cm$^3$, which corresponds to $\approx 1.4$ kg/m$^3$. Considering $h=10$ g/m$^3$ and $\theta=10\%$, our results indicate that the sensor footprint in a forested ecosystem is reduced to 78% or 44% for canopy heights of 15 or 30 m, respectively. Qualitatively, this reduction should be taken into account when calibration or validation of the CRNS probe is performed in forests and in different seasons [e.g., Franz et al., 2013a; Bogena et al., 2013; Lv et al., 2014]. Future investigations should focus on...
various vegetation models and cover a range of parameters in order to gain profound understanding of neutron interactions at individual agricultural or forest sites.

4.4. Penetration Depth in the Soil

The thickness of the probed soil layer is an important advantage of the CRNS method compared to most remote-sensing products. Cosmic-ray neutrons can penetrate the first decimeters of the soil almost unhindered, whereas electromagnetic signals interact within the upper 0–5 cm. Franz et al. [2012b] showed that the effective representation of the penetration depth, \( z'(\theta) \), is a reciprocal function of soil moisture, but it is unclear how it varies with the distance from the probe.

In URANOS, we logged the vertical positions \( d \) where neutrons lost energy in a scattering process, i.e., probed the soil. Above \( \theta \geq 10\% \), the penetration depth of neutrons appears to decrease exponentially. This behavior can be expected from a simple monoenergetic Beer-Lambert approach [Beer, 1852], and has also been found by Zreda et al. [2008]. A simple analytical description of the vertical weighting function was found for \( \theta \geq 10\% \):

\[
W_d(r, \theta) \propto e^{-2d/D_{86}(r, \theta)},
\]

The quantity denotes up to which depth 86\% of the detected neutrons had contact with constituents of the soil. Numerical parameters are provided in Table A1, \( \theta \) is in units of m\(^3\)/m\(^3\).

Figure 8 shows penetration depths \( D_{86}(r, \theta) \) as a function of radial distance \( r \) from the sensor for exemplary soil moisture values \( \theta \). For dry soil, \( D_{86}(r, \theta \approx 1\%) \) ranges from 83 cm right below the sensor to 46 cm at \( r = 300 \) m distance. At most, the penetration depth varies between 15 cm and 83 cm below the sensor for wet and dry soil, respectively. This is in close agreement with depths of 12–76 cm given by Zreda et al. [2008]. The reported values are rather confirmed than contradicted by URANOS, because they stemmed from experiences and various studies in the research field of cosmogenic nuclide production and are thus independent of the mentioned model approach. On average over the first tens of meters distance, the functional dependency on \( \theta \) (equation (8)) is relatively similar to the reciprocal model for the effective sensor depth \( z'(\theta) \) from Franz et al. [2012b]. Their model was constrained on the limits introduced by Zreda et al. [2008] and validated with measurements and hydrodynamic simulations. Further evidence for the correct performance of the URANOS model provides the comparison with measurement depths of 50–100 cm on the Moon or Mars missions, where cosmic-ray neutrons penetrate dry ground of similar chemical composition [Elphic et al., 2008; McKinney et al., 2006].

4.5. Terrain Structures and Topography

Cosmic-ray neutron probes are sometimes placed close to roads, trees, rivers, or in hilly terrains. In an analogous manner, mobile rover surveys inevitably pass alongside forests, lakes, or fields of different land use. In most of these cases, we do not expect an isotropic
footprint of the probe, because large structures of different hydrogen content vary throughout the viewing directions.

In order to quantify the anisotropy of detected neutrons, we simulated four exemplary cases where such scenarios are extreme. In Figure 9, the vicinity of a centered detector is shown and the isotropic footprint \( R_{86}(h, \theta) = 210 \text{ m} \) is indicated (dashed line). Dots illustrate the origin of detected neutrons, where the closest 86% of total neutrons are emphasized (black) in each direction. We discretized the area into 12° sectors in order to quantify range (black dots) and intensity (red) for 30 discrete directions.

In a coast line setup (Figure 9a), the density of the origins (dots) and neutron intensity (red) appear to be much smaller in the ponded area. The range of neutrons decreases by up to 30–40% although neutrons still manage to travel long distances over water. Their contribution to the count rate sharply drops to about 40% at the interface.

In Figure 9b, the detector is placed 50 m away from a 10 m wide river. This setup can be found where cosmic-ray neutron probes are located within small catchments with creeks or irrigated land. Neutron origins clearly show that the river hardly contributes to the signal because most neutrons lose too much energy after probing water (see also point density and neutron intensity for water, Figure 9a). This is also visible in the intensity which shows a slight asymmetry toward the dry side. However, the setup reveals a slightly wider footprint in the direction to the river, as a consequence of the intensity gap.

A detector carried on a dry, concrete road (Figure 9c) is a common scenario for rover applications [e.g., Chrisman and Zreda, 2013]. The sensor detects about 10–20% more neutrons per sector from the road than from other directions. However, the decrease of the footprint along the road due to short-range dominated contribution is marginal. The effect of the road is expected to be weaker for tarry material, as it contains hydrogen and carbon.

In Figure 9d, we illustrate the investigation of neutron detection under more complex topography, here being a 20° steep hill slope. From detailed analysis, we found that the uphill footprint (left) does not differ significantly from downhill (right), although small asymmetries in the neutron origins occur. Neutron intensity from uphill is about 0.26% higher compared to downhill, which is far beyond significance of the count rate.

These idealized cases demonstrate that the geometry of complex terrain only slightly influences soil moisture measurements with the cosmic-ray neutron sensor. However, the anisotropic contributions to the count rate should be investigated individually if accuracy matters. To add more reality to the scenarios, future studies on topography and structures should account for correlating quantities like gradients of air pressure, humidity, or soil moisture down the hill or close to rivers. As a consequence of more collisions and more efficient moderation, these quantities are expected to treat neutrons from uphill preferentially.

4.6. Experimental Evidence?
Since the footprint definition is based on a radial symmetry, direct empirical evidence is difficult to achieve with natural structures. However, approaching water surfaces and transiting the coast line has been a

**Figure 9.** Anisotropy of detected neutron origins (black) and neutron intensity (red) determined for every 12° sector of a circle around a centered detector. The displayed extent is 270 m in radius, whereas the dashed line represents the isotropic footprint with radius \( R_{86}(h, \theta) \approx 210 \text{ m} \), considering \( h = 5 \text{ g/m}^3 \) and \( h = 5\% \). The four exemplary cases illustrate bare soil (white) with (a) a coast line (blue), (b) a 10 m river at 50 m distance, (c) a 10 m concrete road (yellow), and (d) a 20° hill slope from the left down to the right.
common procedure to determine the range of detected neutrons. For example, Kuzhevskij et al. [2003] moved the detector over a lake and interprets that the signal strength is hardly sensitive to neutrons from the land side at distances greater than 200 m. In the last years, many experiments with the COSMOS detector have been performed across a water-land boundary by the group of M. Zreda. First data from Oceanside Pier (California, USA) indicate that the sensitive distance is on the order of 100–200 m at sea level.

With URANOS, we made an attempt to reproduce these transect experiments by moving a 4 m square-shaped detector over pure water and land with exemplary soil moistures from 1 to 30% and fixed air humidity $h = 10 \text{ g/m}^3$. Figure 10 illustrates the simulations and the two experiments mentioned above. Simulated signal strengths clearly correspond to the measurements and give an indication of the soil water content which was unknown at the time of the experiments. The signal gradient is asymmetric over water (left) and land (right), which agrees with results from Franz et al. [2013b], who investigated the influence of large wet structures on the signal strength. It is further interesting to note that $R_{\text{foot}}$ ranges from 168 to 220 m (according to the considered range in soil moisture, 1–30%). However, these values cannot be identified in the experiment, because the signal is almost saturated by 150 m. Both effects can be explained by (1) the overestimation of dry over wet regions in the signal, as a consequence of the nonlinear relation: $\theta \rightarrow N$ [Desilets et al., 2010], (2) the effective removal of traveling neutrons due to the presence of a water body on their way to the detector, and (3) the nonradial geometry of the experiment.

We must conclude that transect experiments do not give a direct measure of the footprint radius under conditions where the instrument is usually applied. However, the presented data provide evidence for the valid performance of the URANOS model.

5. Summary and Conclusions

This work investigates the footprint defined as the water-sensitive support volume of the cosmic-ray neutron sensor. Previous simulations by Desilets and Zreda [2013] drew general conclusions from a number of model assumptions and provided a decent estimate of the footprint for dry conditions. The travel distance of neutrons probing the soil, however, is very sensitive to initial energies and even to small amounts of hydrogen on their way. As a consequence, the complexity of environmental neutron transport appears to impede any attempt to simplify the problem. Therefore, we felt the need for revisiting neutron transport models and for addressing some of the open questions regarding the radial sensitivity, humid climate, or terrain structures. Simulations in this work were performed by the Monte Carlo code URANOS, whereas the concept is applicable to any standard software. From the results of this work, we draw the following conclusions:
1. The revised footprint radius $R_{86}(h, \theta, p, \text{veg})$ is 240 m (18 ha area) for bare soil and purely dry conditions at sea level. However, significant influence of soil moisture $\theta$, humidity $h$ and vegetation can further reduce the radius by more than 40%. In contrast, decreasing air pressure may expand it by $\approx 1\%$ per 10 mbar. The total error $\Delta R_{86}$ was estimated conservatively to be about 4–6%. The dynamic footprint has implications for methods for the interpolation of survey data, irrigation management, and data assimilation for hydrological models.

2. The signal strength per radial distance, $W_r(h, \theta)$, is highly nonlinear in $r$, $h$, and $\theta$ and exhibits extraordinary sensitivity to the first few meters. As a rule of thumb, at least half of the neutron intensity reflected by the soil is due to the first 50 m around the sensor. Consequently, dynamic weighting of horizontal averages can be essential for sensor calibration and validation with soil moisture monitoring networks.

3. The penetration depth $D_{86}(r, \theta)$ of detected neutrons directly below the sensor ranges from $\approx 15$ to 83 cm depending on soil moisture. An exponential decrease with depth is a good estimate for the sensor’s vertical sensitivity, whereas the depth in turn shrinks significantly with radial distance to the sensor.

4. The circular shape of the footprint remains isotropic for most field applications, like hilly terrain, nearby rivers or heterogeneous land. However, large water bodies or forests nearby can reduce range and intensity of detected neutrons from that direction. Dry roads can contribute to an overestimate of neutron counts by a few percent. While rover surveys are often exposed to a variety of environmental conditions, these findings can have implications for interpretation and geostatistical interpolation of spatial data.

5. Transect experiments can be helpful to investigate the detector response to remote water bodies. In the same manner, they allow to validate input models and strategies for Monte Carlo driven simulations. URANOS is able to reproduce these measurements adequately; however, this method is not appropriate to give direct evidence for the radial footprint size.

The present study shows that the description of the footprint and neutron intensity is nontrivial to an exceptional degree. For this reason, it is not possible to conclude with an easy and straight-forward analytical formulation. We recommend to read out values from the figures presented here or from numerical and highly resolved data given in the supporting information. Individual simulations should be performed for every probe in order to analyze the local response specific to the surrounding environment.

Future work is needed to experimentally verify the results of this study. For example, to test the suggested spatial sensitivity of the sensors, the performance of weighted averages needs to be assessed for point data from sampling campaigns or soil moisture monitoring networks. Despite the dynamic characteristics of the footprint, the capability to average water content over a large volume is undisputed and remains a valuable advantage of the method of cosmic-ray neutron sensing in the field of soil moisture monitoring.

### Appendix A: Parameter Functions

The parameter functions $F_i(h, \theta)$ in equation (3) have been obtained empirically. Units for air humidity $h$ and soil moisture $\theta$ are g/m$^3$ and m$^3$/m$^2$, respectively. Table A1 contains the related numerical parameters $p_i$. In the supporting information, we provide application-ready scripts to calculate $W_r$ in R, MatLab and Excel.

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$F_3 = p_0 (1 + p_5 h)e^{-p_1 / (1 + p_5 h)} + p_2 - p_4 \theta$, 
$F_4 = p_0 e^{-p_5 / (1 + p_5 h)} + p_2 - p_4 \theta + p_e h$, 
$F_5 = p_0 \left( \frac{0.02 - 1}{p_1 - p_0 (1 + h - p_5)} \right) \cdot (p_4 - 0)e^{-p_1 / (1 + p_1 h)} + p_2 (0.7 - p_1 h \theta)$, 
$F_6 = p_0 (h + p_1) + p_2 \theta$, 
$F_7 = (p_4 h - p_2 e^{-p_1 / (1 + p_1 h)} + p_2 - p_3 \theta) \cdot (2 + p_1 h)$, 
$F_8 = (p_4 h - p_0 e^{-p_1 / (1 + p_5 h)} + p_2) \cdot (2 + p_2 h)$.

**Appendix B: Weighted Mean for Soil Moisture Comparisons**

Equations (3) and (7) can be used to weight individual point measurements in order to validate or calibrate the signal apparent to a cosmic-ray neutron sensor. The general procedure to obtain a weighted average $\langle \theta_k \rangle$ from measurements $\theta_i$ with the weighting function $W_i$ is as follows:

$$\langle \theta_k \rangle = \frac{\sum_i \theta_i \cdot W_i}{\sum_i W_i}.$$ 

Let $\theta_i$ be a sample of soil moisture at the depth $d_i$ and distance $r_i$ to the cosmic-ray neutron sensor. The field-mean soil moisture $\langle \theta_i \rangle$ and humidity $\langle h \rangle$ can be averaged with the equal weight $W_i = 1 \forall i, j$. We then suggest to first compute the vertical average $\langle \theta_i \rangle$ of the data at each point $i$ with the weighting function $W_i (r_i, \langle \theta_i \rangle)$, equation (7). Second, these values can be averaged horizontally (vl) with the weighting function $W_i (\langle h \rangle, \langle \theta_i \rangle)$, equation (3).

**References**


